Optimal Residential Demand Response in Distribution Networks

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***Abstract*—Demand response (DR) enables customers to adjust their electricity usage to balance supply and demand. Most previous works on DR consider the supply-demand matching in an abstract way without taking into account the underlying power distribution network and the associated power flow and system operational constraints. As a result, the schemes proposed by those works may end up with electricity consumption/shedding decisions that violate those constraints and thus are not feasible. In this paper, we study residential DR with consideration of the power distribution network and the associated constraints. We formulate residential DR as an optimal power flow problem and propose a distributed scheme where the load service entity and the households interactively communicate to compute an optimal demand schedule. To complement our theoretical results, we also simulate an IEEE test distribution system. The simulation results demonstrate two interesting effects of DR. One is the location effect, meaning that the households far away from the feeder tend to reduce more demands in DR. The other is the rebound effect, meaning that DR may create a new peak after the DR event ends if the DR parameters are not chosen carefully. The two effects suggest certain rules we should follow when designing a DR program.**

***Index Terms*—Demand response (DR), distributed algorithms, distribution networks, optimal power flow (OPF), smart grid.**

I. Introduction

DEMAND response (DR) is a mechanism to enable customers to participate in the electricity market in order to improve power system efficiency and integrate renewable generation [1]. Most of the existing DR programs in the United States are for commercial and industrial customers and they have been well studied. Very few DR programs are in use for residential customers [2]. However, as smart grid technologies such as smart metering, smart appliances, and home area network technologies developed significantly over the past years, residential DR becomes increasingly attractive due to its great potential [3].

Residential DR requires the coordination of a large number of households in order to improve the overall power system efficiency and reliability. Such coordination is usually implemented via pricing signals, assuming that customers are price responsive. Extensive algorithms [3]–[8] have been proposed in the literature to determine the prices and customers' responses to the prices. Most of those works consider the supply-demand matching in DR in an abstract way where the aggregate demand is simply equal to the supply. However, households are not isolated with each other, but they are connected by a power distribution network with the associated power flow constraints (e.g., Kirchhoff's laws) and system operational constraints (e.g., voltage tolerances). As a result, the schemes proposed by previous works may end up with electricity consumption/shedding decisions that violate those constraints and thus are not feasible. There are few works which consider DR in direct current (DC) distribution networks [9]. However, they cannot be applied to the most widely-used alternating current (AC) distribution networks.

This paper focuses on the design of a DR scheme for a large number of residential households with consideration of the AC power distribution network and the associated constraints in a smart grid where two-way communications between the load service entity (LSE) and the households are available. More specifically, we consider a direct residential DR program where customers who participate in it sign a contract with the LSE in advance to let the LSE control some of their appliances for a certain period of time. The home energy management systems (HEMs) in the participating households can receive DR control signals from the LSE to coordinate their appliance operations in order to meet the DR objective in a DR event. The objective of the DR is to manage the appliances for each household such that (i) the social welfare (i.e., the customer utilities minus the power losses) is maximized, (ii) the system demand is below a certain limit during peak hours, and (iii) the appliance operational constraints, the power flow constraints, and the system operational constraints are satisfied.

Specifically, we formulate residential DR as an optimal power flow problem (OPF) using a branch flow model [10]. The OPF problem is non-convex due to the power flow constraints and thus is difficult to solve. We relax the problem to be a convex problem. The convex relaxation is not exact in general (i.e., the solution to the relaxed problem is not the same as the solution to the original problem). Recent works [11]–[13] have derived sufficient conditions under which the relaxation is exact for radial networks. Roughly speaking, if the bus voltage is kept around the nominal value and the power injection at each bus is not too large, then the relaxation is exact. For detailed conditions, please refer to [11]. More sufficient conditions can be found in [12], [13]. Those conditions can be checked a priori and hold for a variety of

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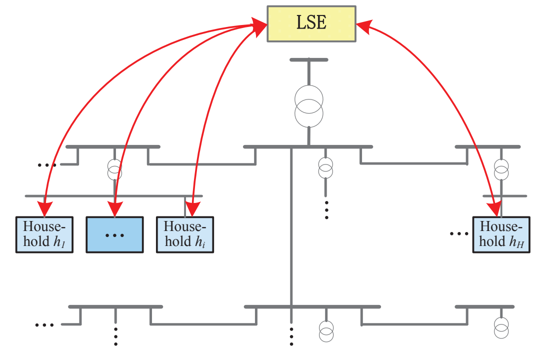
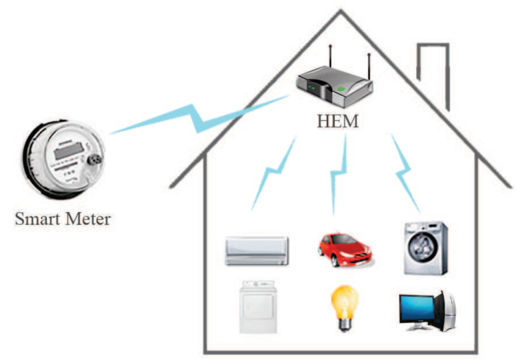
IEEE standard distribution networks and real-world networks. Therefore, we focus on solving the relaxed OPF problem (OPF-r) in this paper. The OPF-r problem is a centralized optimization problem. To solve it in a distributed manner, we propose a DR scheme where the LSE and the HEMs in the households jointly compute an optimal demand schedule. In our proposed DR scheme, the HEM in each household keeps the private information locally (i.e., utility functions and appliance operational constraints) and the LSE has the system information (i.e., network topology, line impedances, power losses, etc.). Therefore, customer privacy (i.e., detailed appliance-level information) is protected in the DR process.

To complement our theoretical model, we apply the proposed DR scheme to an IEEE test radial distribution system [14]. The simulation results demonstrate the effectiveness of our proposed DR scheme and show two interesting effects of DR. One is the location effect meaning that the households far away from the feeder tend to reduce more demands in DR. The other is the rebound effect meaning that DR may create a new peak after the DR event ends if the DR parameters are not chosen carefully. The two effects suggest certain rules we should follow when designing a DR program.

The rest of the paper is organized as follows. We introduce the system model in Section II and propose the DR scheme in Section III. Simulation results and the discussions about the location effect and the rebound effect are provided in Section IV and conclusions are given in Section V.



(a) An LSE's distribution network of H households.



(b) The HEM system in each household.

Fig. 1. An illustration of the system model.

II. System Model

This section describes the system model of the proposed distributed residential DR scheme. We give an overview of the system followed by the appliance model, the customer preference model, the distribution network model, and the DR model. Those models will be used for designing the DR scheme in the following section.

A. System Overview

We consider a residential DR over a distribution network, which is operated by one LSE. In the network, each load bus is connected with a set of households and there are a total of H households $\mathcal{H} := \{h_1, h_2, \dots, h_H\}$ in the system. In each household $h \in \mathcal{H}$, there is a HEM system managing a set of appliances $\mathcal{A}_h := \{a_{h,1}, a_{h,2}, \dots, a_{h,A}\}$ such as air conditioners (ACs), electric vehicles (EVs), dryers, etc. The HEM is also connected with the LSE's communications network via a smart meter so that there is a two-way communication link between the LSE and the household [15]. Since most distribution networks are radial, we focus on only radial distribution networks in this paper. The overall system architecture is shown in Fig. 1.

We use a discrete-time model with a finite horizon in this paper. We consider a time period or namely a scheduling horizon which is divided into T equal intervals Δt, denoted by $\mathcal{T}$. For each appliance $a \in \mathcal{A}_h$, let $p_{h,a}(t)$ and $q_{h,a}(t)$ be the real power and reactive power it draws at time $t \in \mathcal{T}$. The complex power of the appliance can be denoted by $s_{h,a}(t) := p_{h,a}(t) + \mathbf{i}q_{h,a}(t)$. The HEM system in each household is able to gather the power consumption information of the appliances $\{s_{h,a}(t)\}_{a \in \mathcal{A}_h}$ and adjust the electricity usage to achieve certain energy efficiency for the customer. If the customer is enrolled in a DR program, the HEM system can receive DR events issued by the LSE. A DR event would request the participants to shed or reschedule their demands in exchange for some incentives. Although it is possible that customers can change their demands manually, a fully-automated system which can respond to DR events automatically is more favorable for residential customers [16]. To implement such an auto DR system, an intelligent control algorithm is needed for the HEM to manage the operations of the appliances in order to meet the DR objective.

B. Appliance Model

Household appliances can be classified into three types in the context of DR: critical, interruptible, and deferrable loads [6]. Critical loads such as refrigerators, cooking, and critical lighting should not be shifted or shedded at any time. Interruptible loads such as ACs and optional lighting can be shedded during DR. Deferrable loads such as washers, dryers, and EVs can be shifted during DR but they are required to consume a certain minimum energy before deadlines to finish their tasks. Since critical loads cannot participate in DR, we do not consider them here.

For a given appliance $a \in \mathcal{A}_h$, the relationship between the real power and the reactive power is given by the power factor $\eta_{h,a}(t)$:

$$\eta_{h,a}(t) = \frac{p_{h,a}(t)}{|s_{h,a}(t)|}, \forall t \in \mathcal{T}. \quad (1)$$

And we characterize an appliance by a set of constraints on its demand vector $\mathbf{p}_{h,a} := (p_{h,a}(t), t \in \mathcal{T})$.

Now we introduce the appliance operational constraints [7].

- For each appliance, the demand is constrained by a minimum and a maximum power denoted by $p_{h,a}^{\min}(t)$ and $p_{h,a}^{\max}(t)$, respectively:

$$p_{h,a}^{\min}(t) \leq p_{h,a}(t) \leq p_{h,a}^{\max}(t), \forall t \in \mathcal{T}. \quad (2)$$

  Note that $p_{h,a}^{\min}(t)$ and $p_{h,a}^{\max}(t)$ can be also used to set the available working time for the appliance. For example, if the appliance cannot run at time t, then we set $p_{h,a}^{\max}(t) = p_{h,a}^{\min}(t) = 0$.
- For thermostatically controlled appliances such as ACs and heaters, the constraint (2) alone is not enough. To model this kind of appliances, we need to find the relationship between the indoor temperature $T_h^{\text{in}}(t)$ and the demand vector $\mathbf{p}_{h,a}$ (refer to [7] or Section IV for details). We assume that the customer sets a most comfortable temperature $T_h^{\text{comf}}(t)$ and there is a range of temperature that the customer can bear, denoted by $[T_h^{\text{comf,min}}, T_h^{\text{comf,max}}]$. In addition to (2), a thermostatically controlled appliance can be modeled as:

$$T_h^{\text{comf,min}} \leq T_h^{\text{in}}(t) \leq T_h^{\text{comf,max}}, \forall t \in \mathcal{T}. \quad (3)$$

- For deferrable loads, the cumulative energy consumption of the appliances must exceed a certain threshold in order to finish their tasks before deadlines. Let $E_{h,a}^{\min}$ and $E_{h,a}^{\max}$ denote the minimum and maximum total energy that the appliance is required to consume, respectively. The constraint on the total energy consumed by a deferrable load is given by:

$$E_{h,a}^{\min} \leq \sum_{t\in\mathcal{T}} p_{h,a}(t)\Delta t \leq E_{h,a}^{\max}. \quad (4)$$

C. Customer Preference Model

We model customer preference in the DR using the concept of utility function from economics. The utility function $U_{h,a}(\mathbf{p}_{h,a})$ quantifies a customer's benefit or comfort obtained by running an appliance $a \in \mathcal{A}_h$ using its demand vector $\mathbf{p}_{h,a}$. Depending on the type of the appliance, the utility function may take different forms [7].

- For interruptible loads, the utility is dependent on the power it draws at time t and may be time variant if the operation is time sensitive. For example, the utility function for interruptible loads can be defined as:

$$U_{h,a}(\mathbf{p}_{h,a}) := \sum_{t\in\mathcal{T}} U_{h,a}\left(p_{h,a}(t), t\right). \quad (5)$$

- For thermostatically controlled appliances, the utility is related to the temperature $T_h^{\text{in}}(t)$ and the most comfort temperature $T_h^{\text{comf}}(t)$. Therefore, the utility function can be defined in the form of:

$$U_{h,a}(\mathbf{p}_{h,a}) := \sum_{t\in\mathcal{T}} U_{h,a}\left(T_h^{\text{in}}(t), T_h^{\text{comf}}(t)\right). \quad (6)$$

TABLE I
NOTATIONS

$V_i(t)$, $v_i(t)$	complex voltage on bus i with $v_i(t) = \lvert V_i(t)\rvert^2$
$s_i(t) = p_i(t) + \mathbf{i}q_i(t)$	complex load on bus i
$I_{ij}(t)$, $\ell_{ij(t)}$	complex current from buses i to j with $\ell_{ij}(t) = \lvert I_{ij}(t)\rvert^2$
$S_{ij}(t) = P_{ij}(t) + \mathbf{i}Q_{ij}(t)$	complex power from buses i to j
$z_{ij} = r_{ij} + \mathbf{i}x_{ij}$	impedance on line (i, j)

- For deferrable loads, since a customer mainly concerns if the task can be finished before deadline, we define the utility as a function of the total energy consumption:

$$U_{h,a}(\mathbf{p}_{h,a}) := U_{h,a}\left(\sum_{t\in\mathcal{T}} p_{h,a}(t)\Delta t\right). \quad (7)$$

For the rest of the paper, we assume that the utility function $U_{h,a}(\mathbf{p}_{h,a})$ is a continuously differentiable concave function for all $h \in \mathcal{H}, a \in \mathcal{A}_h$.

D. Distribution Network Model

A power distribution network can be modeled as a connected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where each node $i \in \mathcal{N}$ represents a bus and each link in $\mathcal{E}$ represents a branch (line or transformer). The graph $\mathcal{G}$ is a tree for radial distribution networks. We denote a branch by $(i, j) \in \mathcal{E}$. We index the buses in $\mathcal{N}$ by $i = 0, 1, \ldots, n$, and bus 0 denotes the feeder which has a fixed voltage and flexible power injection. Table I summarizes the key notations used in modeling distribution networks for the ease of reference.

For each branch $(i, j) \in \mathcal{E}$, let $z_{ij} := r_{ij} + \mathbf{i}x_{i,j}$ denote the complex impedance of the branch, $I_{ij}(t)$ denote the complex current from buses i to j, and $S_{ij}(t) := P_{ij}(t)+\mathbf{i}Q_{ij}(t)$ denote the complex power flowing from buses i to j.

For each bus $i \in \mathcal{N}$, let $V_i(t)$ denote the complex voltage at bus i and $s_i(t) := p_i(t) + \mathbf{i}q_i(t)$ denote the complex bus load. Specifically, the feeder voltage V_0 is fixed and given. $s_0(t)$ is the power injected to the distribution system. Each load bus $i \in \mathcal{N} \setminus \{0\}$ supplies a set of households which are connected to the bus denoted by $\mathcal{H}_i \subset \mathcal{H}$. The aggregate load at each bus satisfies:

$$s_i(t) = \sum_{h\in\mathcal{H}_i} \sum_{a\in\mathcal{A}_h} s_{h,a}(t), \forall i \in \mathcal{N} \setminus \{0\}, \forall t \in \mathcal{T}. \quad (8)$$

Given the radial distribution network $\mathcal{G}$, the feeder voltage V_0, and the impedances $\{z_{ij}\}_{(i,j)\in\mathcal{E}}$, then the other variables including the power flows, the voltages, the currents, and the bus loads satisfy the following physical laws for all branches $(i, j) \in \mathcal{E}$ and all $t \in \mathcal{T}$.

- Ohm's law:

$$V_i(t) - V_j(t) = z_{ij}I_{ij}(t); \quad (9)$$

- Power flow definition:

$$S_{ij}(t) = V_i(t)I_{ij}^*(t); \quad (10)$$

- Power balance:

$$S_{ij}(t) - z_{ij}\lvert I_{ij}(t)\rvert^2 - \sum_{k:(j,k)\in\mathcal{E}} S_{jk}(t) = s_j(t). \quad (11)$$

Using equations (9)–(11) and in terms of real variables, we have [17]: $\forall (i,j) \in \mathcal{E}, \forall t \in \mathcal{T}$,

$$p_j(t) = P_{ij}(t) - r_{ij}\ell_{ij}(t) - \sum_{k:(j,k)\in\mathcal{E}} P_{jk}(t), \tag{12}$$

$$q_j(t) = Q_{ij}(t) - x_{ij}\ell_{ij}(t) - \sum_{k:(j,k)\in\mathcal{E}} Q_{jk}(t), \tag{13}$$

$$v_j(t) = v_i(t) - 2\left(r_{ij}P_{ij}(t) + x_{ij}Q_{ij}(t)\right) + (r_{ij}^2 + x_{ij}^2)\ell_{ij}(t), \tag{14}$$

$$\ell_{ij}(t) = \frac{P_{ij}(t)^2 + Q_{ij}(t)^2}{v_i(t)}, \tag{15}$$

where $\ell_{ij}(t) := |I_{ij}(t)|^2$ and $v_i(t) := |V_i(t)|^2$.

Equations (12)–(15) define a system of equations in the variables $(\mathbf{P}(t), \mathbf{Q}(t), \mathbf{v}(t), \mathbf{l}(t), \mathbf{s}(t))$, where $\mathbf{P}(t) := (P_{ij}(t), (i,j) \in \mathcal{E})$, $\mathbf{Q}(t) := (Q_{ij}(t), (i,j) \in \mathcal{E})$, $\mathbf{v}(t) := (v_i(t), i \in \mathcal{N} \setminus \{0\})$, $\mathbf{l}(t) := (\ell_{ij}(t), (i,j) \in \mathcal{E})$, and $\mathbf{s}(t) := (s_i(t), i \in \mathcal{N} \setminus \{0\})$. The phase angles of the voltages and the currents are not included. But they can be uniquely determined for radial distribution networks [10].

E. DR Model

The objective of the LSE is to deliver reliable and high-quality power to the customers through the distribution network. However, during peak hours, the system demand may exceed the capacity or the LSE may need to use expensive generations to guarantee reliability. The voltages may also deviate significantly from their nominal values, which reduces power quality. Thus, in this paper, we study DR aiming at keeping the system demand under a certain limit while meeting the voltage tolerance constraints during peak hours.

A DR event can be defined by a schedule and a demand limit $(\mathcal{T}_d, s_{\max})$, where $\mathcal{T}_d \subseteq \mathcal{T}$ is the schedule which specifies the start time and the end time of the DR event and $s_{\max}$ is the demand limit imposed by either the system capacity or the LSE according to the supply.

Given the DR event, the system demand constraint can be modeled as:

$$|s_0(t)| \le s_{\max}, \forall t \in \mathcal{T}_d, \tag{16}$$

where $s_0(t)$ is the total complex power injected to the distribution system and it is given by:

$$s_0(t) = \sum_{j:(0,j)\in\mathcal{E}} S_{0j}(t), \forall t \in \mathcal{T}. \tag{17}$$

We also consider the voltage tolerance constraints in the distribution network which keep the magnitudes of the voltage at each load bus within a certain range during a DR event:

$$V_i^{\min} \le |V_i(t)| \le V_i^{\max}, \ \forall i \in \mathcal{N} \setminus \{0\}, \forall t \in \mathcal{T}_d. \tag{18}$$

The allowed voltage range for different distribution systems can be found in the standard [18].

The objective of the proposed DR scheme is to find a set of optimal demand vectors to maximize the aggregate utilities of the appliances in the households and minimize the power losses in the distribution network subject to the appliance operational constraints, the power flow constraints, the system demand constraint, and the system operational constraints (voltage tolerances).

We define $\mathbf{P} := (\mathbf{P}(t), t \in \mathcal{T})$, $\mathbf{Q} := (\mathbf{Q}(t), t \in \mathcal{T})$, $\mathbf{v} := (\mathbf{v}(t), t \in \mathcal{T})$, $\mathbf{l} := (\mathbf{l}(t), t \in \mathcal{T})$, $\mathbf{s}_{h,a} := (s_{h,a}(t), t \in \mathcal{T})$, and $\mathbf{s} := (\mathbf{s}_{h,a}, h \in \mathcal{H}, a \in \mathcal{A}_h)$. The residential DR can be formulated as an OPF problem.

OPF:

$$\max_{\mathbf{P},\mathbf{Q},\mathbf{v},\mathbf{l},\mathbf{s}} \quad \sum_{h\in\mathcal{H}} \sum_{a\in\mathcal{A}_h} U_{h,a}(\mathbf{p}_{h,a}) - \kappa \sum_{t\in\mathcal{T}} \sum_{(i,j)\in\mathcal{E}} r_{ij}\ell_{ij}(t)$$
$$\text{s.t.} \quad (1)-(4), (8), (12)-(18),$$

where $U_{h,a}(\mathbf{p}_{h,a})$ is defined by (5)–(7); (1)–(4) are the appliance operational constraints; (8), (12)–(15) are the power flow constraints; (16) and (17) are the system demand constraints; (18) is the voltage tolerance constraint; and κ is a parameter to trade off between customer utility maximization and power loss minimization. A large κ means that the LSE is more self-interested in minimizing the power losses rather than maximizing the customer utilities.

III. Distributed DR Scheme

In this paper, we focus on developing a scalable distributed DR scheme rather than a centralized scheme due to the large number of appliances that need to be managed by the scheme. Moreover, the proposed DR scheme also needs to protect the privacy for the residential customers. To design such a distributed DR scheme, we relax the previous OPF problem to be a convex problem and propose a distributed algorithm to solve it. The convexity of the relaxed OPF problem guarantees the convergence of the distributed algorithm.

A. Convexification of OPF

The previous OPF problem is non-convex due to the quadratic equality constraint in (15) and thus is difficult to solve. Moreover, most decentralized algorithms require convexity to ensure convergence [19]. We therefore relax them to inequalities:

$$\ell_{ij}(t) \ge \frac{P_{ij}(t)^2 + Q_{ij}(t)^2}{v_i(t)}, \ \forall (i,j) \in \mathcal{E}, \forall t \in \mathcal{T}. \tag{19}$$

Now we consider the following convex relaxation of OPF.

OPF-r:

$$\max_{\mathbf{P},\mathbf{Q},\mathbf{v},\mathbf{l},\mathbf{s}} \quad \sum_{h\in\mathcal{H}} \sum_{a\in\mathcal{A}_h} U_{h,a}(\mathbf{p}_{h,a}) - \kappa \sum_{t\in\mathcal{T}} \sum_{(i,j)\in\mathcal{E}} r_{ij}\ell_{ij}(t)$$
$$\text{s.t.} \quad (1)-(4), (8), (12)-(14), (16)-(19).$$

OPF-r provides an upper bound to OPF. For an optimal solution of OPF-r, if the equality in (19) is attained at the solution, then it is also an optimal solution of OPF. We call OPF-r an exact relaxation of OPF if every solution to OPF-r is also a solution of OPF, and vice versa.

The sufficient conditions under which OPF-r is an exact relaxation of OPF for radial distribution networks have been derived in previous works [11]–[13]. Roughly speaking, if the bus voltage is kept around the nominal value and the power injection at each bus is not too large, then the relaxation is exact. For detailed conditions, please refer to [11]. More sufficient conditions can be found in [12], [13]. Those conditions are verified to hold for many IEEE standard distribution networks and real-world networks. When OPF-r is an exact relaxation of OPF, we can focus on solving the convex optimization problem OPF-r. In this paper, we assume that the conditions for exact relaxation of OPF to OPF-r specified in [11]–[13] hold for the radial distribution network and therefore OPF-r is an exact relaxation of OPF and strong duality holds for OPF-r.

B. Distributed Algorithm

To solve OPF-r in a centralized way, it requires not only the distribution network information but also the private information of the appliances (i.e., utility functions and schedules). In order to protect customer privacy and make the DR scalable, we propose a distributed DR scheme to solve the OPF-r problem using the predictor corrector proximal multiplier (PCPM) algorithm (refer to [20] or the appendix for details).

Initially set $k \leftarrow 0$. The HEM in each household $h \in \mathcal{H}$ sets the initial demand schedule $\mathbf{s}_{h,a}^k$ for each appliance $a \in \mathcal{A}_h$ according to its preferable demand schedule. The HEM then communicates its aggregate demand schedule $\mathbf{s}_h^k := \sum_{a\in\mathcal{A}_h} \mathbf{s}_{h,a}^k$ to the LSE. In the meantime, the LSE randomly chooses the initial $s_i^k(t) := p_i^k(t) + \mathbf{i}q_i^k(t)$ and two virtual control signals $\{\mu_i^k(t)\}_{t\in\mathcal{T}}$, $\{\lambda_i^k(t)\}_{t\in\mathcal{T}}$ for each bus $i \in \mathcal{N}\setminus\{0\}$.

At the beginning of the k-th step, the LSE sends two DR control signals $\hat{\mu}_i^k(t) := \mu_i^k(t) + \gamma\left(\sum_{h\in\mathcal{H}_i} p_h^k(t) - p_i^k(t)\right)$ and $\hat{\lambda}_i^k(t) := \lambda_i^k(t) + \gamma\left(\sum_{h\in\mathcal{H}_i} q_h^k(t) - q_i^k(t)\right)$ to the HEMs in households $h \in \mathcal{H}_i$ for all $t \in \mathcal{T}$, where γ is a positive constant. Then,

- The HEM in each household $h \in \mathcal{H}_i$ solves the following problem for each appliance $a \in \mathcal{A}_h$.

  DR-household:

$$\begin{aligned}\max_{\mathbf{s}_{h,a}} \quad & U_{h,a}(\mathbf{p}_{h,a}) - (\hat{\boldsymbol{\mu}}_i^k)^T\mathbf{p}_{h,a} - (\hat{\boldsymbol{\lambda}}_i^k)^T\mathbf{q}_{h,a} \\ & -\frac{1}{2\gamma}||\mathbf{p}_{h,a} - \mathbf{p}_{h,a}^k||^2 - \frac{1}{2\gamma}||\mathbf{q}_{h,a} - \mathbf{q}_{h,a}^k||^2 \\ \text{s.t.} \quad & (1)-(4),\end{aligned}$$

  where $\hat{\boldsymbol{\mu}}_i^k := (\hat{\mu}_i^k(t), t \in \mathcal{T})$ and $\hat{\boldsymbol{\lambda}}_i^k := (\hat{\lambda}_i^k(t), t \in \mathcal{T})$. The optimal $\mathbf{s}_{h,a}^*$ is set as $\mathbf{s}_{h,a}^{k+1}$.
- The LSE solves the following problem for each time $t \in \mathcal{T}$.

  DR-LSE:

$$\begin{aligned}\max_{\substack{\mathbf{P}(t),\mathbf{Q}(t),\\ \mathbf{v}(t),\mathbf{l}(t),\mathbf{s}(t)}} \quad & (\hat{\boldsymbol{\mu}}^k(t))^T\mathbf{p}(t) + (\hat{\boldsymbol{\lambda}}^k(t))^T\mathbf{q}(t) \\ & -\kappa\sum_{(i,j)\in\mathcal{E}} r_{ij}\ell_{ij}(t) - \frac{1}{2\gamma}||\mathbf{p}(t) - \mathbf{p}^k(t)||^2 \\ & -\frac{1}{2\gamma}||\mathbf{q}(t) - \mathbf{q}^k(t)||^2 \\ \text{s.t.} \quad & (12)-(14),(16)-(19),\end{aligned}$$

  where $\hat{\boldsymbol{\mu}}^k(t) := (\hat{\mu}_i^k(t), i \in \mathcal{N}\setminus\{0\})$ and $\hat{\boldsymbol{\lambda}}^k(t) := (\hat{\lambda}_i^k(t), i \in \mathcal{N}\setminus\{0\})$. The optimal $\mathbf{s}^*(t)$ is set as $\mathbf{s}^{k+1}(t)$.

Algorithm 1 - The Proposed Distributed DR Scheme.

1: **initialization** $k \leftarrow 0$. The HEM sets the initial $\mathbf{s}_{h,a}^k$ and returns the aggregate demand schedule $\mathbf{s}_h^k$ to the LSE. The LSE sets the initial $\mu_i^k(t)$, $\lambda_i^k(t)$ and the initial $s_i^k(t)$ randomly.
2: **repeat**
3: The LSE updates $\hat{\mu}_i^k(t)$ and $\hat{\lambda}_i^k(t)$ and sends the DR control signals $\hat{\boldsymbol{\mu}}_i^k$ and $\hat{\boldsymbol{\lambda}}_i^k$ to the HEMs in the households $h \in \mathcal{H}_i$.
4: The HEM in each household calculates a new demand schedule $\mathbf{s}_{h,a}^{k+1}$ for each appliance $a \in \mathcal{A}_h$ by solving the DR-household problem.
5: The LSE computes a new $\mathbf{s}^{k+1}(t)$ for each time $t \in \mathcal{T}$ by solving the DR-LSE problem.
6: The HEM communicates the aggregate demand schedule $\mathbf{s}_h^{k+1}$ to the LSE.
7: The LSE updates $\mu_i^{k+1}(t)$ and $\lambda_i^{k+1}(t)$.
8: $k \leftarrow k + 1$.
9: **until** convergence

At the end of the k-th step, the HEM in household h communicates its aggregate demand schedule $\mathbf{s}_h^{k+1} := \sum_{a\in\mathcal{A}_h} \mathbf{s}_{h,a}^{k+1}$ to the LSE and the LSE updates $\mu_i^{k+1}(t) := \mu_i^k(t)+\gamma\left(\sum_{h\in\mathcal{H}_i} p_h^{k+1}(t) - p_i^{k+1}(t)\right)$ and $\lambda_i^{k+1}(t) := \lambda_i^k(t)+\gamma\left(\sum_{h\in\mathcal{H}_i} q_h^{k+1}(t) - q_i^{k+1}(t)\right)$ for all $i \in \mathcal{N}\setminus\{0\}$ and all $t \in \mathcal{T}$. Set $k \leftarrow k+1$, and repeat the process until convergence.

A complete description of the proposed DR scheme can be found in Algorithm 1. When γ is small enough, the above algorithm will converge to the optimal solution of OPF-r which is also the optimal solution of OPF if the relaxation is exact, and $\left(\sum_{h\in\mathcal{H}_i} p_h^k(t) - p_i^k(t)\right)$ and $\left(\sum_{h\in\mathcal{H}_i} q_h^k(t) - q_i^k(t)\right)$ will converge to zero [20]. As we can see, the LSE and the HEMs in the households interactively communicate to compute the optimal demand schedule. Therefore, the two-way communications network in the distribution system is crucial to implement the proposed DR scheme. Notice that after the LSE and the customers jointly compute the optimal demand schedule over $t \in \mathcal{T}$, the LSE only controls the demands during the DR period according to the optimal schedule over $t \in \mathcal{T}_d$. The customers may or may not follow the optimal demand schedule exactly for the rest of the time $\mathcal{T}\setminus\mathcal{T}_d$.

In the proposed DR scheme, the private information of the customer including the utility functions $U_{h,a}(\mathbf{p}_{h,a})$ and the appliance operational constraints (1)–(4) appears only in the DR-household problem which is solved by the HEM owned by the customer. The LSE solves the DR-LSE problem using the system information including the power flow constraints (12)–(14) and (19), the system demand constraints (16) and (17), the voltage tolerance constraints (18), and the power losses $\sum_{(i,j)\in\mathcal{E}} r_{ij}\ell_{ij}(t)$. Therefore, there is no appliance-level information gathered by the LSE and customer privacy can be protected in the DR process.

IV. Performance Evaluation

In this section, we demonstrate the proposed DR scheme by applying it to an IEEE standard distribution system. We first describe the distribution system used in the simulation and give the parameters of the scheme. Then we present the simulation results of the proposed DR scheme and discuss the interesting effects that we observe from the simulation.

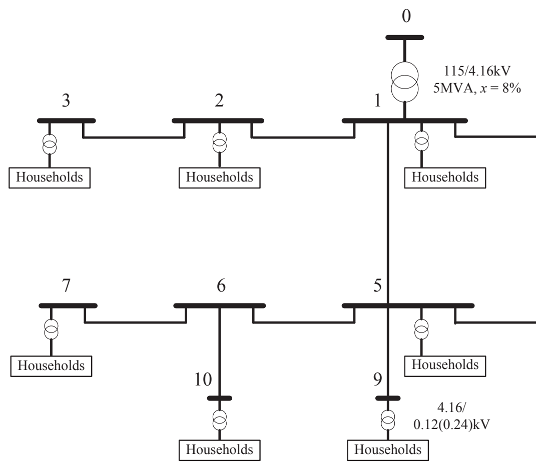


Fig. 2. The modified IEEE standard distribution system.

A. Simulation Setup

We use the IEEE 13-node test feeder [14] as the power distribution system which is shown in Fig. 2.[1] We assume that there are 10 households connected to each load bus. In the simulation, a day starts from 8 am. The time interval Δt in the model is one hour and we denote a day by $\mathcal{T}_D := \{8, 9, \dots, 24, 1, \dots, 7\}$ where each $t \in \mathcal{T}_D$ denotes the hour of $[t, t+1]$. The scheduling horizon $\mathcal{T}$ used by the LSE to calculate the optimal DR strategy is chosen to be $\mathcal{T} := \{t_s, t_s + 1, \dots, 7\} \subseteq \mathcal{T}_D$, where t_s is the time that the DR event starts.

A total of 6 different appliances including ACs, EVs, washers, dryers, lighting, and plug loads are considered in the simulation. The power factor of each appliance $\eta_{h,a}(t)$ is assumed to be constant and its value is selected randomly from $[0.8, 0.9]$. We further assume that there is a preferable demand schedule (i.e., the baseline power consumption without any DR incentives) for each appliance denoted by $\mathbf{p}^{\text{pref}}_{h,a} := (p^{\text{pref}}_{h,a}(t), t \in \mathcal{T})$. Detailed descriptions for the appliances are given as follows.

1) ACs: An AC is a thermostatically controlled appliance. Let $T^{\text{out}}_h(t)$ denote the outside temperature. We assume that the indoor temperature evolves according to [7]:

$$T^{\text{in}}_h(t) = T^{\text{in}}_h(t-1) + \alpha \left(T^{\text{out}}_h(t) - T^{\text{in}}_h(t-1) \right) + \beta p_{h,a}(t), \tag{20}$$

where α and β are the thermal parameters of the environment and the appliance, respectively. α is a positive constant and β is positive if the AC is running in the heating mode or negative in the cooling mode. Using (20), we define the utility of an AC as $U_{h,a}\left(T^{\text{in}}_h(t), T^{\text{comf}}_h(t)\right) := c_{h,a} - b_{h,a}\left(T^{\text{in}}_h(t) - T^{\text{comf}}_h(t)\right)^2$, where $b_{h,a}$ and $c_{h,a}$ are positive constants.

[1] In order to exemplify the effect of DR on both the households and the distribution network, we made several changes to the standard IEEE 13-node test feeder: the inline transformer between node 633 and node 634 is omitted, the switch between node 671 and node 692 is closed, and the line lengths are increased by 5 times. The feeder has a nominal voltage of 4.16kV. Since our focus is on residential customers, we assume that there is a secondary distribution transformer at each load bus which scales the voltage down to 120/240V to serve multiple households.

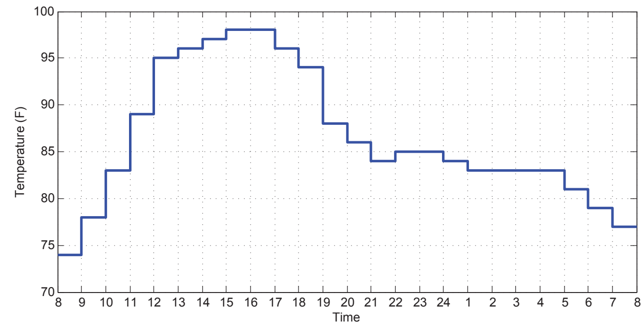


Fig. 3. Outside temperature of a day.

In the simulation, we choose the thermal parameters as $\alpha = 0.9$ and β is chosen randomly from $[-0.008, -0.005]$. The outside temperature of the day is given in Fig. 3 which is a typical summer day in Southern California. For each household, we assume the comfortable temperature range to be $[70\text{F}, 79\text{F}]$ and the most comfortable temperature $T^{\text{comf}}_h(t)$ is chosen randomly from $[73\text{F}, 76\text{F}]$. The maximum and minimum power are $p^{\max}_{h,a} = 4\text{kW}$ and $p^{\min}_{h,a} = 0\text{kW}$, respectively.

2) EVs: An EV is a deferrable load. We assume that the EV arriving time $t_{h,e}$ is randomly chosen from $[17, 19]$. It starts charging immediately after arriving and must finish charging before $t = 6$. The maximum and minimum charging rates are $p^{\max}_{h,a} = 3\text{kW}$ and $p^{\min}_{h,a} = 0\text{kW}$, respectively. The maximum charging requirement $E^{\max}_{h,a}$ is chosen randomly from $[20\text{kWh}, 24\text{kWh}]$ and $E^{\min}_{h,a}$ is chosen randomly from $[15\text{kWh}, 18\text{kWh}]$. The utility function is in the form of $U_{h,a}(\mathbf{p}_{h,a}) := b_{h,a}\left(\sum_{t\in\mathcal{T}} p_{h,a}(t)\right) - \sum_{t\in\mathcal{T}} t|p_{h,a}(t) - p^{\text{pref}}_{h,a}(t)| + c_{h,a}$.

3) Washers: A washer is a deferrable load. Its starting time $t_{h,w}$ is chosen randomly from $[t_{h,e}, 20]$. It must finish its job within 2 hours. The maximum and minimum power are $p^{\max}_{h,a} = 700\text{W}$ and $p^{\min}_{h,a} = 0\text{W}$, respectively. The maximum energy requirement $E^{\max}_{h,a}$ is chosen randomly from $[900\text{Wh}, 1200\text{Wh}]$ and $E^{\min}_{h,a}$ is chosen randomly from $[600\text{Wh}, 800\text{Wh}]$. The utility function takes the same form as that of an EV.

4) Dryers: A dryer is a deferrable load. It starts working at $t_{h,w} + 2$ and must finish before $t = 1$. The maximum and minimum power are $p^{\max}_{h,a} = 5\text{kW}$ and $p^{\min}_{h,a} = 0\text{kW}$, respectively. The maximum energy requirement $E^{\max}_{h,a}$ is chosen randomly from $[7.5\text{kWh}, 10\text{kWh}]$ and $E^{\min}_{h,a}$ is chosen randomly from $[4\text{kWh}, 5\text{kWh}]$. The utility function takes the same form as that of an EV.

5) Lighting: Lighting is an interruptible load. Its working time is $[19, 24] \cup [1, 7]$. The maximum and minimum power are $p^{\max}_{h,a} = 1.0\text{kW}$ and $p^{\min}_{h,a} = 0.5\text{kW}$, respectively. The utility function takes the form of $U_{h,a}(p_{h,a}(t), t) := c_{h,a} - b_{h,a}(p_{h,a}(t) - p^{\text{pref}}_{h,a}(t))^2$.

6) Plug Loads: Plug loads include other common household appliances such as TVs, home theaters, PCs, etc. They belong to interruptible loads. The maximum and minimum power are $p^{\max}_{h,a} = 500\text{W}$ and $p^{\min}_{h,a} = 0\text{W}$, respectively. The utility function takes the same form as that of lighting.

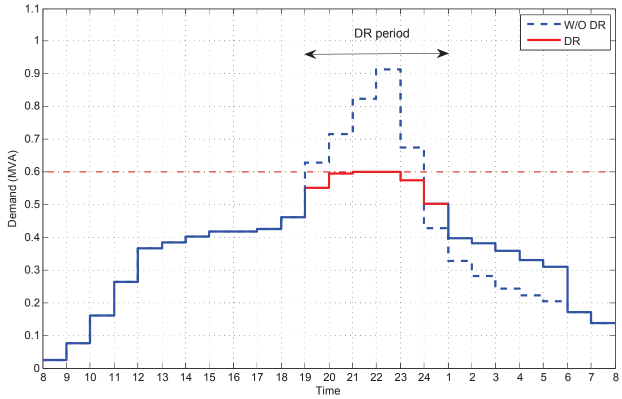


Fig. 4. Load profile of the feeder $|s_0(t)|$ without and with DR.

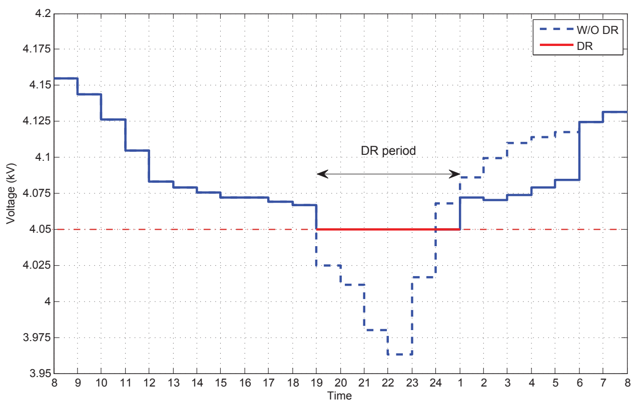


Fig. 5. Minimum bus voltage profile without and with DR.

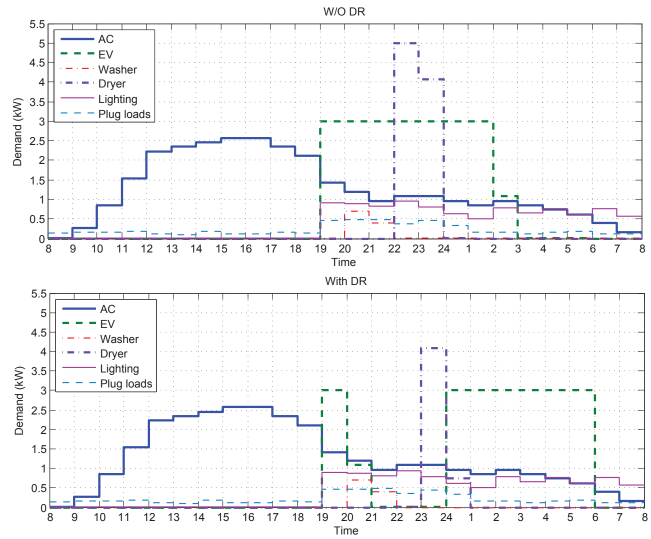


Fig. 6. Load profile of the appliances in one of the households without and with DR.

B. Case Study

We simulate our proposed DR scheme in the modified IEEE standard distribution system. The voltage at the feeder V_0 is assumed to be fixed at 4.16kV and there are no voltage regulators or capacitors on the distribution lines. The minimum allowed voltage at each load bus $V_i^{\min}$ is set to be 4.05kV [18]. The parameters in our proposed DR scheme are chosen as $\kappa := 0.01$ and $\gamma := 0.25$.

We use the preferable schedules of the appliances as the baseline in the simulation. More specifically, the AC keeps the indoor temperature to the most comfortable temperature $T_h^{\text{comf}}(t)$ all day. The EV, the washer, and the dryer run at their maximum power $p_{h,a}^{\max}(t)$ until the maximum energy requirement $E_{h,a}^{\max}$ is met. Lighting and the plug loads use the power as they request.

The load profile of the feeder $|s_0(t)|$ without DR is shown by the dashed line in Fig. 4. It can be seen that the system demand is low for most of the day. The peak starts at $t = 19$ and lasts until $t = 23$. The dashed line in Fig. 5 shows the minimum bus voltage in the distribution network over time. It can be seen that the minimum bus voltage is below the voltage rating during the peak hours. By comparing Fig. 4 with Fig. 5, we can find that there is a significant correlation between the load level and the voltage drop. The higher the demand is, the more significant the voltage drop is.

To simulate a DR event, we need to choose the DR parameters including the demand limit $s_{\max}$ and the schedule $\mathcal{T}_d$. In our simulation, we assume that the LSE imposes a demand limit of $s_{\max} = 0.6\text{MVA}$ during the time period $[19, 24]$. The DR period is chosen in a way to prevent the rebound effect which will be discussed later.

The simulated load profile of the feeder $|s_0(t)|$ with DR is shown by the solid line in Fig. 4. Note that the LSE only controls the demands during the peak hours (shown by red). The load profile after the DR ends is based on the optimal demand schedules produced by our proposed DR scheme. The customers may or may not follow the optimal demand schedules. From Fig. 4, it can be seen that our proposed DR scheme can effectively manage the appliances of the households in the distribution network to keep the system demand under the demand limit during the DR event. The solid line in Fig. 5 shows the minimum bus voltage profile with DR. We can see that in addition to keep the system demand below the limit, our proposed DR scheme is also able to maintain the bus voltage levels within the allowed range during the DR event.

Fig. 6 shows the load profile of the appliances in one of the households without and with DR. Both load shifting and load shedding can be found in the figure: the deferrable loads (the EV and the dryer) are shifted and the total energy that the dryer consumes is reduced. If we compare the daily system demand without and with DR, we can find a demand reduction of 0.05MVA which is about 3% of the daily system demand.

Fig. 7 and Fig. 8 show the dynamics of the proposed distributed DR scheme. As we can see from the figures, both DR-household and DR-LSE converge fast in the simulation. For all the simulations, we also verify that the solution to the centralized OPF-r problem is the same as the solution to the distributed algorithm using the CVX package [21]. We further verify that the equality in (19) is attained in the optimal solution to OPF-r, i.e., OPF-r is an exact relaxation of OPF.

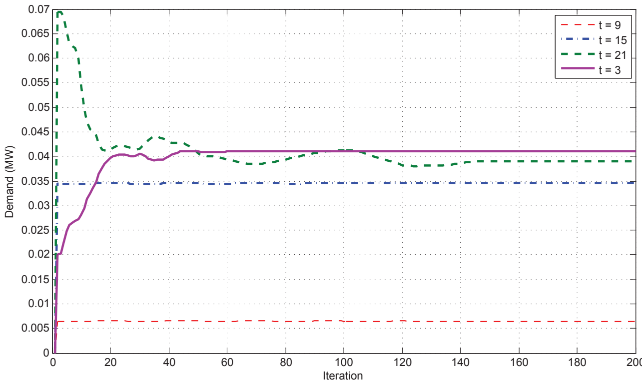


Fig. 7. Dynamics of DR-household: the aggregate real power at bus 8.

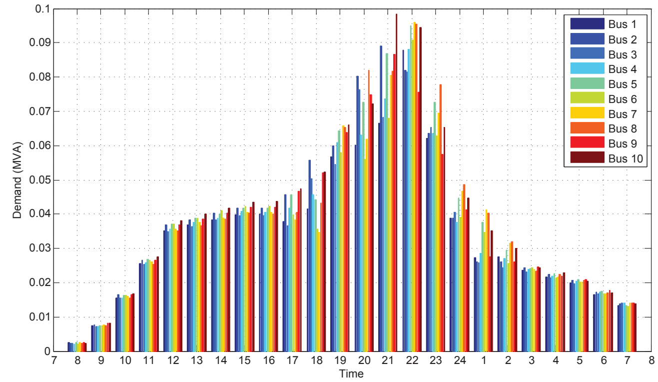


Fig. 9. Aggregate load profile of the households at each bus without DR.

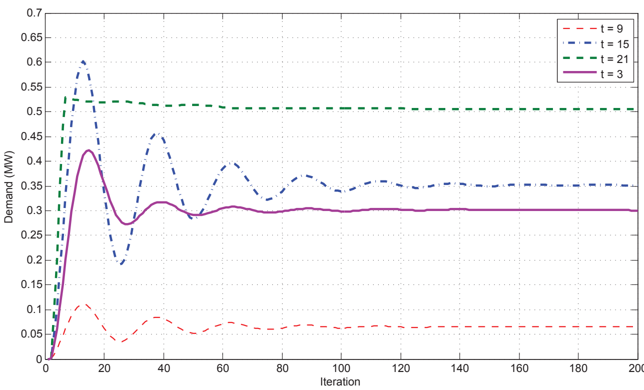


Fig. 8. Dynamics of DR-LSE: the real power injected to the system p_0.

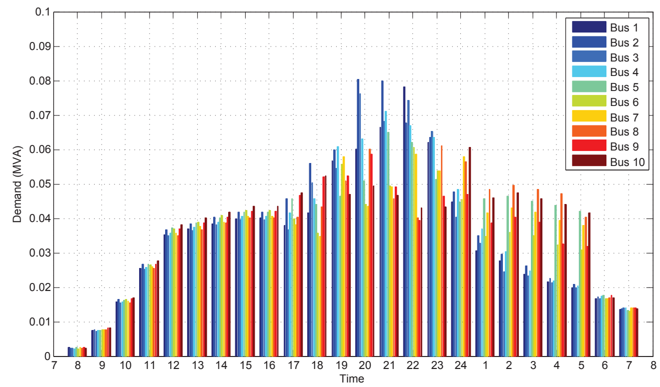


Fig. 10. Aggregate load profile of the households at each bus with DR.

C. Discussions

1) Location Effect: Fig. 9 and Fig. 10 show the aggregate load profile of the households at each load bus $|s_i(t)|$ without and with DR, respectively. By comparing the two figures, it can be found that the loads at the buses far away from the feeder (buses 5–10) contribute more than the loads at the buses close to the feeder (buses 1–4) to the demand reduction in the DR event. The shifted demands from on-peak hours to off-peak hours are largely from the buses far away from the feeder.

The reason for this location effect is due to both the power loss minimization and the voltage regulation in DR. The power loss and voltage drop along the distribution line are related to not only the load level but also the length of the line. As the length of the distribution line increases, the impedance of the line increases, leading to a higher power loss and voltage drop. Therefore, in order to decrease the total power injection into the distribution system, which includes both the total power consumption and the power losses, and also to meet the voltage tolerance constraints, the households at the buses far away from the feeder must shed or shift more demands than the households at the buses close to the feeder. This location effect implies a potential fairness issue in DR since the impacts of DR on the households are not the same. The LSE may need to set the DR incentives given to the households differently based on their locations in the network. Mechanisms to compensate such location discrimination can be developed in the future.

2) Rebound Effect: In the previous simulation, we use the DR schedule $[19, 24]$. An interesting rebound effect of DR can be observed if we reduce the DR period by one hour. The simulation result using the new schedule $[19, 23]$ is shown in Fig. 11. It can be seen from the figure that although our proposed DR scheme is effective to keep the system demand below the demand limit during the DR period $[19, 23]$, it creates a rebound peak about 0.8MVA at $t = 24$ right after the DR event ends. The rebound effect is not desirable because the new peak brings the same problems to the system as the old peak.

The reason for this rebound effect is that when the DR shifts the peak demands to off-peak periods, it may create another peak. The rebound effect shown in our simulation suggests that the LSE should choose the DR parameters (i.e., the demand limit and the DR schedule) carefully when designing a DR event. Since the demand limit is usually determined by the system capacity and the power supply, the freedom of the design lies mainly in the DR schedule. Both the load profile and the voltage profile need to be considered when determining the DR schedule because the time needed for the demand reduction and the voltage regulation may not be the same. A protection time period may also be needed in the DR schedule to prevent the rebound effect. In our pervious simulation, the protection period is one hour. Heuristic methods can be developed for the LSE to set the length of the protection period in the future.

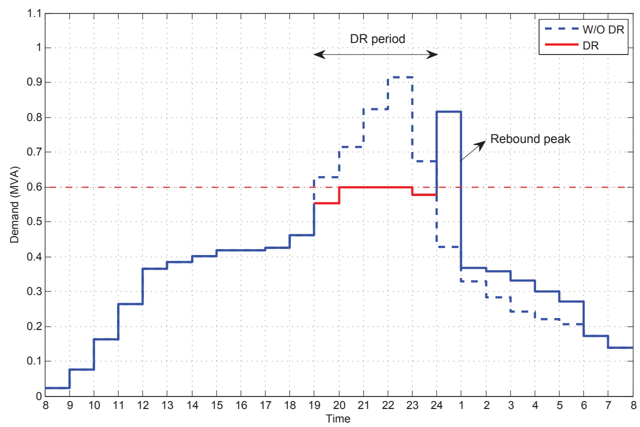


Fig. 11. Rebound effect of DR.

V. Conclusions

In this paper, we study residential DR with consideration of the underlying AC power distribution network and the associated power flow and system operational constraints. This residential DR is modeled as an OPF problem. We then relax the non-convex OPF problem to be a convex problem and propose a distributed DR scheme for the LSE and the households to jointly compute an optimal demand schedule. Using an IEEE test distribution system as an illustrative example, we demonstrate two interesting effects of DR. One is the location effect, meaning that the households far away from the feeder tend to reduce more demands in DR. The other is the rebound effect, meaning that DR may create a new peak after the DR event ends if the DR parameters are not chosen carefully. The two effects suggest certain rules we should follow when designing a DR program. Future work includes designing compensation mechanisms for the location discrimination and heuristic methods to deal with the rebound effect.

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Appendix
Introduction to PCPM

In this paper, we develop a distributed DR scheme using the predictor corrector proximal multiplier (PCPM) algorithm [20]. PCPM is a decomposition method for solving convex optimization problem. At each iteration, it computes two proximal steps in the dual variables and one proximal step in the primal variables. We give a very brief description of the PCPM algorithm below.

Consider a convex optimization problem with separable structure of the form:

$$\min_{\mathbf{x}\in\mathcal{X},\mathbf{y}\in\mathcal{Y}} \quad f(\mathbf{x}) + g(\mathbf{y}) \tag{21}$$

$$\text{s.t.} \quad A\mathbf{x} + B\mathbf{y} = \mathbf{c}. \tag{22}$$

Let $\mathbf{z}$ be the Lagrangian variable for the constraint (22).

The steps of the PCPM algorithm to solve the problem are given as follows:

1) Initially set $k \leftarrow 0$ and choose the initial $(\mathbf{x}^0, \mathbf{y}^0, \mathbf{z}^0)$ randomly.
2) For each $k \geq 0$, update a virtual variable $\hat{\mathbf{z}}^k := \mathbf{z}^k + \gamma(A\mathbf{x}^k + B\mathbf{y}^k - \mathbf{c})$ where $\gamma > 0$ is a constant step size.
3) Solve
$$\mathbf{x}^{k+1} = \arg\min_{\mathbf{x}\in\mathcal{X}}\{f(\mathbf{x}) + (\hat{\mathbf{z}}^k)^T A\mathbf{x} + \frac{1}{2\gamma}||\mathbf{x} - \mathbf{x}^k||^2\},$$
$$\mathbf{y}^{k+1} = \arg\min_{\mathbf{y}\in\mathcal{Y}}\{g(\mathbf{y}) + (\hat{\mathbf{z}}^k)^T B\mathbf{y} + \frac{1}{2\gamma}||\mathbf{y} - \mathbf{y}^k||^2\}.$$
4) Update $\mathbf{z}^{k+1} := \mathbf{z}^k + \gamma(A\mathbf{x}^{k+1} + B\mathbf{y}^{k+1} - \mathbf{c})$.
5) $k \leftarrow k + 1$, and go to step 2 until convergence.

It has been shown in [20] that the above algorithm will converge to a primal-dual optimal solution $(\mathbf{x}^*, \mathbf{y}^*, \mathbf{z}^*)$ for a sufficient small positive step size γ as long as strong duality holds for the convex problem (21).

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