Optimal Sizing and Placement of Battery Energy Storage in Distribution System Based on Solar Size for Voltage Regulation

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Abstract This paper proposes a new strategy to achieve voltage regulation in distributed power systems in the presence of solar energy sources and battery storage systems. The goal is to find the minimum size of battery storage and its corresponding location in the network based on the size and place of the integrated solar generation. The proposed method formulates the problem by employing the network impedance matrix to obtain an analytical solution instead of using a recursive algorithm such as power flow. The required modifications for modeling the slack and PV buses (generator buses) are utilized to increase the accuracy of the approach. The use of reactive power control to regulate the voltage regulation is not always an optimal solution as in distribution systems \( R/X \) is large. In this paper the minimum size and the best place of battery storage is achieved by optimizing the amount of both active and reactive power exchanged by battery storage and its grid-tie inverter (GTI) based on the network topology and \( R/X \) ratios in the distribution system. Simulation results for the IEEE 14-bus system verify the effectiveness of the proposed approach.

Index Terms – Battery Storage, Solar, Distributed System, Micro Grid, Optimal Placement, Minimum Sizing.

I. INTRODUCTION

Electric utility companies are concerned about potential difficulties due to the large scale penetration of photovoltaics (PVs) in the power grid. One of the challenges that may stem from the high penetration level of solar PVs and supply-demand imbalances is reverse power flow which may lead to voltage rise and variation. This issue will especially affect the distribution side of the grid where household PVs are connected [1]-[3]. In addition, voltage variation due to uncontrollable nature of PVs can have a direct impact on the operation of load tap changers, voltage-controlled capacitor banks, and line voltage regulators, which may cause additional step-voltage changes. Based on the delays (30-90s) in the control of the existing devices, minute-based step-voltage variations may be experienced. In addition, more frequent operation of these devices leads to more maintenance requirements, which consequently makes the expected life cycle of these devices short. Therefore methods to improved voltage regulation have been developed to overcome the issue of overvoltage [4]-[6]. However, in distribution side and Low Voltage (LV) feeders the impedance is mostly resistive and the \( R/X \) ratio is considerable, as opposed to medium voltage feeder and transmission lines. Thus, the role of active power can be as important as reactive power on voltage variations and losses in a radial LV distribution feeder. This fact justifies the investigation to solve the voltage regulation problem by employing both active and reactive power and finding a tradeoff between these two parameters to optimally regulate the voltage [7]. One of the practical and effective solutions to achieve a flexible real power control and solve the overvoltage issue is the deployment of energy storage systems [8]. Among all feasible types of energy storage technologies, a battery based energy storage system can be considered as widely used and fairly developed. As a result, batteries in the form of battery energy storage systems (BESS) and its application in electric vehicles (EVs) are being actively researched in the field of distributed generation [9]-[11]. The application of battery storage system for voltage regulation and peak load shaving is investigated in previous works [12]-[13]. It is shown in [4] how the BESS could help devices such as on-load tap changer and step voltage regulator to mitigate the overvoltage through reducing or eliminating the burden from these devices. The optimized size and best location of BESS has not been investigated in these researches. To expand the application of BESS in distribution systems, research on BESS size optimization and placement is important and beneficial for cost justification of their commercial applications in distribution systems.

Different algorithms have been proposed to minimize the size of battery storage system in [7], [14]-[17]. While many of them focus on reducing cost or increasing revenue of bulk energy arbitrage [14]-[15], some aim to compensate for intermittent nature and power fluctuation of renewable energy such as solar and wind [16]-[17]. However, these studies have not concentrated on battery storage system capability to improve the voltage regulation of the supply side of the distribution system and reduced workload on tap changer and voltage regulator. In addition, the optimal placement of battery storage system in distribution systems to affect maximum nodes voltage is yet a research topic to be addressed.

In this paper, a new strategy is presented to solve the overvoltage and undervoltage issues in a distribution system with solar energy penetration. Based on the location of solar energy and amount of penetration, the size and place of battery storage system are obtained such that a BESS with minimum size is able to improve the voltage profile at maximum number of nodes in an optimal way. This approach benefits from active and reactive power control for voltage regulation and based on the topology of distribution system the optimum ratio of exchanging active and reactive power by BESS and grid-tie inverter is calculated. In this method first the power system is presented in terms of impedance matrix which is a fast and powerful tool for power system analysis. Then the minimum transferring active and reactive power through BESS and inverter is formulated as a function of solar energy penetration, and location of solar, battery and

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compensated node. For this purpose, new techniques are introduced to model the slack and PV buses, during impedance matrix analysis, to increase the accuracy of the proposed approach. In contrast to the approach in [7]-[18], in the proposed approach, instead of using load flow analysis which is a recursive algorithm, the modified impedance matrix analysis is employed which leads to an analytical solution. The size and place of BESS is directly obtained from the optimization using the developed functions compared with the time consuming scanning approaches.

The rest of the paper is organized as follows. In Section II, the principles of the proposed algorithm are described. Then, simulation results obtained from applying the proposed algorithm on IEEE 14-bus systems are presented in Sections III. Finally, concluding remarks are in Sections IV.

II. METHODOLOGY

Grid integration of BESS through grid-tie inverter helps to improve the voltage profile at that bus as well as other buses in the network depending on the topology of the system. When a solar energy source injects power into the system, voltages in the network change depending on the location of the solar sources and impedances of the system. The problem arises when the voltage of one or several buses exceed the standard limit at high PV penetration level, or drop under standard limit at high load level. The focus of this paper is to find the minimum size and best place for BESS to improve the voltage profile at the maximum possible number of nodes. This will allow benefitting from the maximum capacity of solar while reducing the overvoltage effect at high penetration levels.

Although reactive power control is assumed to be the most significant method for voltage improvement, the $R/X$ ratio of the distribution feeder limits its effectiveness. That means, the reactive compensation alone is not sufficient to achieve the desired voltage profile and energy storage is also required for this task. As a result, there should be an optimum way to combine active and reactive power based on $R/X$ of the network to achieve the desired voltage regulation for each node. This paper is aiming to find that optimum point from the developed mathematical functions.

Develop the impedance matrix of the system ($Z$ matrix) makes it easy to calculate the voltage change caused by PV and BESS penetration at each bus [19]. However, the impedance matrix of the system does not reflect the effect of slack bus which has always constant voltage and PV buses with constant voltage amplitude and active power. This paper utilizes a method to add the impact of the slack and PV buses on other buses in the impedance matrix. Note that, the advantages of this approach over power flow analysis are first, the impedance matrix analysis calculates the voltage changes in each node avoiding any recursive algorithm so it is faster. Second, it allows developing the voltage changes of each bus as a function of penetrated current at other buses. Therefore, it makes it easier to analytically find the relationship between location of BESS and solar and amount of exchanging power with specific node voltage. Finding this mathematical relationship helps with solving the optimization problem.

A. Problem Statement

Suppose that $V_{k, S, B}$ represents the measured voltage at bus $k$ affected by solar and BESS penetration where $B$ is defined as $B = \{ ( j_\text{bess}, P_\text{bess}, Q_\text{bess} ) \}$ including the bus number, active and reactive power of BESS, respectively, and $S$ is defined as $S = \{ ( i_{pv}, P_{pv} ) \}$ containing the bus number to which the solar is integrated $(i_{pv})$, and amount of injected active power to the grid $(P_{pv})$, respectively. Here, it is assumed that the solar grid-tie inverter is controlled such that the solar always inject active power under unity power factor to take the maximum benefit from the capacity of solar energy and GTI. Define $\Theta_{i_{pv}} = \{ V_{k, S, B} \leq j_{bess} \leq n, 0 \leq P_{pv} \leq P_{max}, P_{bess} \text{and } Q_{bess} \in \Re \}$ as a set of all possible $V_{k, S, B}$ for specific $i_{pv}$ where $n$ is the total number of network busses. In addition, define $\Phi_{j_{bess}} \subseteq \Theta_{i_{pv}}$, where its members satisfy $V_{min} \leq V_{k, S, B} \leq V_{max}$ with the minimum pair of $(P_{bess}, Q_{bess})$ and integrating BESS at node $j_{bess}$, a subset of $\Theta_{i_{pv}}$ that can uniquely identify minimum $(P_{bess}, Q_{bess})$ that regulate the voltage of node $k$ in the network when BESS is at node $j_{bess}$. Having $\Phi_{j_{bess}}$, the target is first to find the minimum unique $(P_{bess}, Q_{bess})$ that can regulate the voltage of maximum possible node in the network when BESS is in node $j_{bess}$ (optimal solution for BESS at $j_{bess}$) and second, to find the best location $(j_{bess})$ for BESS to provide voltage regulation for the maximum possible node but with less $(P_{bess}, Q_{bess})$.

B. Slack, PV, and PQ bus effects

The bus impedance matrix provides important information regarding the power system network, which can be used to advantage in network calculations [19]. The bus voltages corresponding to the initial values $V^0$ of the bus current can be obtained from $V^0 = Z_{bus}I^0$. When the bus currents are changed from the initial values to new $I^0 + \Delta I$, the new bus voltages can be obtain from the superposition equation

$$V = Z_{bus}(I^0 + \Delta I) = Z_{bus}I^0 + Z_{bus}\Delta I = V^0 + \Delta V$$

where $\Delta V$ expresses the bus voltage changes from their original values. Therefore, by having the initial voltages of the network $V^0$ at steady state from power flow, the voltage changes caused by adding renewable energy sources can be easily calculated from impedance matrix equation (1) which is the focus of this work. However, the challenge is that in the real power system the voltage and angle of the slack bus are always constant which are not considered in this analysis. As a result the obtained results from this approach are not quite accurate. In order to overcome this challenge and model the effect of slack bus in impedance matrix analysis the new concept of $f_{\text{mirror}}$ current is defined here. Suppose that current $I_i$ models the impact of a new added renewable source at bus $i$, then the network voltage changes $(\Delta V)$ can be calculated as:
\[
\Delta V_i = \begin{bmatrix} Z_{11} & \ldots & Z_{1n} & 0 \\
\vdots & & \vdots & \vdots \\
Z_{ni} & \ldots & Z_{nn} & 0 \\
\end{bmatrix} \begin{bmatrix} I_1 \\
\vdots \\
I_n \\
\end{bmatrix}
\]

where \( \{Z_{11} , \ldots , Z_{nn}\} \) are the elements of impedance matrix and \( n \) is the number of buses. Equation (2) depicts that the voltage change of slack bus, normally the first bus is considered as slack bus \( (\Delta V_1) \), is governed by \( \Delta V_1 = Z_{ii} I \). While this voltage should not change. Therefore, the \( \delta \) mirror current \( I_1 = -(Z_{ii}/Z_{11})I_1 \) is added to compensate the effect of \( I_1 \) and keep the \( V_1 \) constant. That is, for each new source current integrated to the network during the analysis, the \( \delta \) mirror current term should be added to the first current element \( (I_1) \) to cancel out its impact on the slack bus voltage.

The same challenge exists for PV buses. In PV buses the active power and voltage amplitude are not changing. Therefore, adding a new source should not affect the former parameters. However, modeling the PV buses in impedance matrix makes the analysis complicated. Here the proposed approach is applying current sources at those nodes such that the total resultant of existing current sources in the network changes neither voltage amplitude nor voltage angle. Then the active power can be modeled as constant impedance since by having constant voltage, the active power remains constant. Although this method ignores the voltage angle variation and consequently reactive power changing at PV buses, provides a reasonable accuracy through a simple approach. The third category of buses is PQ buses. The PQ buses which normally represent the load buses in the system can be modeled as constant impedances during the development of impedance matrix. Although PQ buses are known as constant active (P) and reactive (Q) loads, in this application the target is regulating the voltage so the constant impedance load \( (Z) \) can perfectly model this type of buses.

C. Current source modeling of solar source

In impedance matrix analysis in order to evaluate the voltage changes due to any modification of the network, those modifications should be reflected either in the impedance matrix or in the current vector. For example in case of adding/removing loads or lines the impedance matrix is affected. However, when a new source is added to the system, the impact of this source should be modeled as a current source in current vector. In case of solar energy source, the grid-tie inverter is normally controlled to deliver maximum power under unity power factor [20]. Therefore, a solar energy source can be expressed as amount of active power generation. However, the proposed approach is looking for the current corresponding to the active power. Having constant active power the amount of solar current depends on the voltage of solar connected node while this voltage is a function of injected current. Suppose that the first node of the network is slack bus and there are \( m \) PV buses in the network. Therefore, referring to section B description, adding the solar current at bus \( i \) leads to compensation currents for PV buses (bus 2 to bus \( m \)) and their mirror currents for slack bus.

\[
\Delta V_i = \begin{bmatrix} Z_{i1} & \ldots & Z_{in} & 0 \\
\vdots & & \vdots & \vdots \\
Z_{ni} & \ldots & Z_{nn} & 0 \\
\end{bmatrix} \begin{bmatrix} I_1 \\
\vdots \\
I_n \\
\end{bmatrix}
\]

Therefore, the solar voltage and current equation can be obtained as

\[
V_i = v_i^0 + (Z_{i2} - Z_0) \frac{Z_{12}}{Z_{11}} I_1 + \cdots + (Z_{in} - Z_0) \frac{Z_{1n}}{Z_{11}} I_n + (Z_{ii} - Z_0) \frac{Z_{1i}}{Z_{11}} I_1
\]

where \( v_i^0 \) is initial voltage of node \( i \) before adding solar panel and it is known. Referring to the description in section B, Currents \( I_2, \ldots, I_n \) are calculated as a function of \( I_1 \) such that voltages of buses 2 to \( m \) remain unchanged.

\[
\begin{bmatrix}
(Z_{e2} - Z_{e1}) I_1 \\
(Z_{e3} - Z_{e1}) I_1 \\
\vdots \\
(Z_{en} - Z_{e1}) I_1 \\
\end{bmatrix} = \begin{bmatrix}
(Z_{e2} - Z_{e1}) \frac{Z_{12}}{Z_{11}} \\
(Z_{e3} - Z_{e1}) \frac{Z_{13}}{Z_{11}} \\
\vdots \\
(Z_{en} - Z_{e1}) \frac{Z_{1n}}{Z_{11}} \\
\end{bmatrix} I_1
\]

Finding \( I_2, \ldots, I_n \) from (5) and substitute in (4) all elements on the right side of (4) are in terms of \( I_1 \). For operation under unity power factor

\[
I_1 = S^* V_i^* = P_{pv}/V_i^*
\]

where \( S^* \) represents apparent power conjugate and \( V_i^* \) is conjugate of the solar node voltage. By replacing (6) in (4) and solving the equation (4) for \( V_i \), the \( I_1 \) corresponding to the desired solar \( P_{pv} \) can be obtained.

D. Optimal battery sizing

After modifying the impedance matrix to model the effects of slack, PV, and PQ buses and integrating the solar energy as a current source based on its injected active power, the voltage changes of each node can be obtained easily as a linear function of solar current through \( \Delta V_{ki} = Z_{eqi} I_i \) where the coefficient \( Z_{eqi} \) is an appropriate combination of impedance matrix elements. The same function can be developed for the BESS grid-tie inverter current connected at node \( j_{\text{bess}} \) \( (\Delta V_{kj} = Z_{eqj} I_j) \). As a result the voltage of each node affected by both solar and battery storage system can be calculated as:

\[
V_k = V_k^0 + Z_{eqi} I_i + Z_{eqj} I_j
\]

Assume that the maximum and minimum allowable voltage are \( V_{\text{max}} \) and \( V_{\text{min}} \). Then the optimum direction for bringing back the voltage from overvoltage and undervoltage zones to the acceptable range is aligned with the voltage vector \( V_k^0 + Z_{eqi} I_j \) as it satisfies the minimum distance. As a result the minimum required current injected by battery storage to compensate overvoltage and undervoltage is calculated in (8) and (9), respectively.

\[
l_j^{\text{min}} = \frac{\left( V_k^0 + Z_{eqj} I_j - V_{\text{max}} \right)}{Z_{eqj}} \times \frac{\left( V_k^0 + Z_{eqj} I_j \right)}{V_k^0 + Z_{eqj} I_j}
\]
Developing the minimum BESS current as a function of solar current provides a useful tool to control the BESS based on solar current feedback to improve voltage profile by minimum effort.

E. Best Placement

In section D the minimum BESS current for improving the voltage of each node is calculated. Therefore, the proposed algorithm ends up with \( n \) (number of buses) minimum currents. Each current has both imaginary and real element. Therefore, the minimum resultant current \( (I_{j})_{\text{min}} \) that can satisfy the voltage regulation for all nodes is composed of maximum real part and maximum imaginary part among all currents \( (I_{j})_{k} \)

\[
(I_{j})_{\text{min}} = \max\{\text{real}((I_{j})_{\text{min}})\}_{0 \leq k < n} + j \cdot \max\{\text{image}((I_{j})_{\text{min}})\}_{0 \leq k < n}
\]

(10)

\( (I_{j})_{\text{min}} \) is the minimum current that contains all \( n \) current \( (I_{j})_{k} \) components required for each node voltage regulation. It should be noted that a BESS is able to improve either undervoltage or overvoltage issue at each time because the direction of required current for each task is different. Therefore, based on the number of nodes which are either in undervoltage zone or overvoltage zone, the function of BESS can be determined. In addition, there is a possibility that the BESS cannot regulate the voltage of all nodes, in such a case the nodes have less priority are eliminated from procedure of finding the minimum resultant current \( (I_{j})_{\text{min}} \).

Finally the proposed algorithm shown in Fig. 1 based on obtained result from placement of BESS at each node find the best BESS location to improve the voltage profile of maximum nodes with minimum battery sizing. This algorithm checks the number of nodes that BESS, connected at node \( j \), improves their voltage profile and by changing the location of BESS, among all locations which lead to maximum regulated nodes finds the minimum current.

III. SIMULATION RESULTS

The IEEE 14-bus benchmarks [21] are considered as the case studies as shown in Figs. 2.

The proposed algorithm is applied to find the minimum BESS size besides its best place. The obtained results from proposed algorithm are applied to the power load flow to validate their accuracy.

In first scenario, without loss of generality the solar source is placed at node 5 in IEEE 14-bus while its power varies from 0 to 1pu with resolution of 0.1pu. The BESS is designed to regulate the node voltages for whole range of solar power variation. In addition, referring to ANSI C84.1 the acceptable voltage tolerance range is considered as \( \pm 5\% \). The results are shown in Table I. As illustrated in Table I, the best location for BESS is bus number 9 while 0.1196pu multiple by the desired time of voltage regulation at maximum solar penetration (1pu) defines the size of BESS. In addition, overrated inverters (0.3849pu) for additional capacity of reactive power control is proposed as minimum size of inverter to regulate all bus voltages in the network. At each instant of time the BESS and GTI active and reactive power can be calculated from (8) as a function of solar penetrations.

![Fig. 1. Algorithm of finding the best BESS placement](image)

The voltage profile shown in Table I, with 0.2pu \( P_{pv} \) resolution, validates the regulated voltage in range of \( 1 \pm 0.05 \text{pu} \). Note that the voltage of bus 1 and 2 (slack and PV bus) are always constant.

In second scenario, again 1pu solar is applied to the system but this time the total 1pu power is distributed between nodes 5 to 14. Since the nodes geographically are close to each other, it is assumed that all of them receive the same radiation and they have the identical capacity (0.1pu). The other conditions are as same as scenario one. This scenario can represent integration of solar to a neighborhood houses. Results are shown in Table II. In this scenario the algorithm proposes bus 7 as the best location for BESS while less active power (0.0845pu) and more reactive power (0.4338pu) is required for voltage regulation.
In this paper, a new method for designing a BESS in distribution systems is developed. This method allows calculating the optimal size and place of BESS based on the size of integrated solar in distributed system with the goal of voltage regulation. The optimal BESS current is formulated as a function of solar current by employing modified impedance matrix analysis. Then, the required power from BESS to improve voltage profile is evaluated for all possible BESS locations. The proposed approach by utilizing the active power control along with the reactive power control improves the capability of voltage regulation in distribution system. The future work would be adding the effect of load levels change.

VI. CONCLUSIONS

ACKNOWLEDGMENT

This work has been sponsored in part by grant from the LADWP/DOE fund 20699 & 20686, Smart Grid Regional Demonstration Project.

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