A Distributed Optimal Energy Management Strategy for Microgrids

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***Abstract*—Energy management in microgrids is typically formulated as a non-linear optimization problem. Solving it in a centralized manner not only requires high computational capabilities at the microgrid central controller (MGCC) but may also infringe customer privacy. Existing distributed approaches, on the other hand, assume that all the generations and loads are connected to one bus and ignore the underlying power distribution network and the associated power flows and system operational constraints. Consequently, the schedules produced by those algorithms may violate those constraints and thus are not feasible in practice. Therefore, the focus of this paper is on the design of a distributed energy management strategy (EMS) for the optimal operation of microgrids with consideration of the distribution network and the associated constraints. Specifically, we formulate microgrid energy management as an optimal power flow problem and propose a distributed EMS where the MGCC and the local controllers jointly compute an optimal schedule. As one demonstration, we apply the proposed distributed EMS to a real microgrid in Guangdong Province, China, consisting of photovoltaics, wind turbines, diesel generators, and a battery energy storage system. The simulation results demonstrate that the proposed distributed EMS is effective in both islanded and grid-connected mode. It is also shown that the proposed algorithm converges fast.**

I. Introduction

A microgrid is a low-voltage distribution system consisting of distributed energy resources (DERs) and controllable loads, which can be operated in either islanded or grid-connected mode [1]. DERs include a variety of distributed generation (DG) units such as wind turbines (WTs) and photovoltaics (PVs) and distributed storage (DS) units such as batteries. Sound operation of a microgrid requires an energy management strategy (EMS) which controls the power flows in the microgrid by adjusting the power imported/exported from/to the main grid, the dispatchable DERs, and the controllable loads based on the present and forecasted market information, generation, and load, respectively, in order to meet certain operational objectives (e.g., minimizing costs) [1].

Energy management in microgrids is typically formulated as a non-linear optimization problem. Various centralized methods have been proposed to solve it in the literature, including mixed integer programming [2], sequential quadratic programming [3], neural networks [4], etc. The centralized approaches [2]–[4] require high computational capabilities at the microgrid central controller (MGCC), which is neither efficient nor scalable. Moreover, a centralized EMS requires the MGCC to gather information of the DERs (e.g., production costs, constraints, etc.) and the loads (e.g., customer preferences, constraints, etc.) as the inputs for optimization. However, different DERs may belong to different entities and they may keep their information private [5]. Customers may also be unwilling to expose their information due to the issue of privacy [6]. Therefore, in this paper, we are interested in developing a distributed EMS which is efficient, scalable, and privacy preserving.

Several distributed algorithms have been proposed for the operation of microgrids in the literature. In [5], a distributed algorithm based on the classical symmetrical assignment problem is proposed. Energy management is formulated as a resource allocation problem in [7] and distributed algorithms are proposed for distributed allocation. A convex problem formulation can be found in [8] and dual decomposition is used to develop a distributed EMS to maintain the supply-demand balance in microgrids. A privacy-preserving energy scheduling algorithm in microgrids is proposed in [6], where the privacy constraints are integrated with the linear programming model and distributed algorithms are developed.

The problem with the existing distributed approaches [5]–[8] is that they consider the supply-demand matching in an abstract way, where the aggregate demand is simply equal to the supply. They assume that all the generations and loads are connected to one bus and ignore the underlying power distribution network and the associated power flows (e.g., Kirchhoff's law) and system operational constraints (e.g., voltage tolerances). Consequently, the schedules produced by those algorithms may violate those constraints and thus are not feasible in practice. It is worth noting that distribution networks have been taken into account in a few recent demand response studies [9]. However, the idea of integrating distribution networks with distributed energy management where both supply side and demand side management (DSM) are considered has not been explored.

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The focus of this paper is on the design of a distributed EMS for the optimal operation of microgrids with consideration of the underlying power distribution network and the associated constraints. More specifically, we consider a microgrid consisting of multiple DERs and controllable loads. The objective of the EMS is to control the power flows in the microgrid in order to i) maximize the use of renewable DERs and minimize the costs of generation, the costs of energy storage, and the costs of energy purchase from the main grid, ii) minimize the dissatisfactions of the customers in the DSM, and iii) minimize the power losses subject to the DER constraints, the load constraints, the power flow constraints, and the system operational constraints.

Specifically, we formulate energy management in microgrids as an optimal power flow (OPF) problem. The OPF problem is difficult to solve due to the non-convex power flow constraints. We convexify the OPF problem by relaxing the power flow constraints (See [10], [11] for a tutorial on convex relaxation of OPF). Sufficient conditions for the exactness of the relaxation have been derived in recent works [12]–[14], which hold for a variety of IEEE test systems and real distribution systems. Therefore, we focus on solving the relaxed OPF problem (OPF-r) in this paper. The OPF-r problem is a centralized convex optimization problem. To solve it in a distributed manner, we propose a distributed EMS where the MGCC and the local controllers (LCs) jointly compute an optimal schedule.

As one demonstration, we apply the proposed distributed optimal EMS to a real microgrid in Guangdong Province, China, consisting of PVs, WTs, diesel generators, and a battery energy storage system (BESS). The simulation results demonstrate the effectiveness of the proposed EMS in both islanded and grid-connected mode. It is also shown that the proposed distributed algorithm converges fast.

The rest of the paper is organized as follows. We introduce the system model in Section II and propose the EMS in Section III. Simulation results are provided in Section IV and conclusions are given in Section V.

II. System Model

In this section, we describe the system model for developing the proposed distributed EMS. We first give an overview of the system followed by the detailed models of the DG, the DS, and the loads considered in the microgrid. We then model the power distribution network using a branch flow model and formulate microgrid energy management as an OPF problem.

A. System Overview

A low-voltage power distribution network generally has a radial structure [9]. Thus, we consider a radial microgrid consisting of a set of DG units denoted by $\mathcal{G} \triangleq \{g_1, g_2, \ldots, g_G\}$, DS units denoted by $\mathcal{B} \triangleq \{b_1, b_2, \ldots, b_B\}$, and controllable loads denoted by $\mathcal{L} \triangleq \{l_1, l_2, \ldots, l_L\}$. In the microgrid, there is a MGCC which coordinates the operation of the DERs and the controllable loads. At each of the DERs and the loads, there is a LC which is able to coordinate with the MGCC to compute its schedule locally via a two-way communication infrastructure. Fig. 1 shows the system architecture.

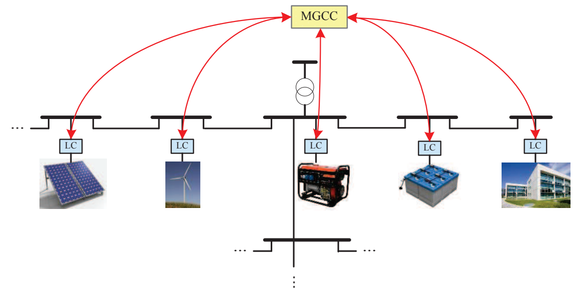

Fig. 1. System architecture.

In this paper, we use a discrete-time model with a finite horizon. We consider a time period or namely a scheduling horizon which is divided into T equal intervals Δt, denoted by $\mathcal{T} \triangleq \{0, 1, \ldots, T-1\}$. By changing the length of the scheduling horizon, we can consider both day-ahead and real-time operations of the microgrid.

B. DG Model

We consider three types of DG units in the microgrid: PVs, WTs, and diesel generators, where PVs and WTs are non-dispatchable renewable DERs and diesel is dispatchable. For each DG $g \in \mathcal{G}$, we denote its complex output power by $s_g(t) \triangleq p_g(t) + \mathbf{i}q_g(t)$, where $p_g(t)$ is the active power and $q_g(t)$ is the reactive power. The detailed models of DG units are given as follows.

1) PV: Given the sun irradiance $r_g(t)$, the output power of a PV unit g at time t can be modeled as [15]:

$$p_g(t) = \sigma_g A_g r_g(t), \forall t \in \mathcal{T}, \tag{1}$$

where σ_g is the efficiency and A_g is the PV area.

2) WT: Given the wind speed $v(t)$, the output power of a WT unit g at time t can be approximately modeled as [15]:

$$p_g(t) = \begin{cases} p_{gr} \frac{v(t)-v_{gi}}{v_{gr}-v_{gi}}, & v_{gi} \le v(t) \le v_{gr} \\ p_{gr}, & v_{gr} \le v(t) \le v_{go} \\ 0, & \text{otherwise} \end{cases}, \forall t \in \mathcal{T}, \tag{2}$$

where v_{gi} is the cut-in wind speed, v_{gr} is the rated wind speed, v_{go} is the cut-off wind speed, and p_{gr} is the rated output power.

3) Diesel: Diesel is dispatchable so its output power is a variable with the following constraints:

$$0 \le p_g(t) \le p_g^{\max}, \forall t \in \mathcal{T}, \tag{3}$$

where $p_g^{\max}$ is the maximum output power.

For a given DG unit $g \in \mathcal{G}$, its reactive power is bounded by:

$$q_g^{\min} \le q_g(t) \le q_g^{\max}, \forall t \in \mathcal{T}, \tag{4}$$

where $q_g^{\min}$ and $q_g^{\max}$ are the minimum and maximum reactive power, respectively.

We model the diesel generation cost at each time $t \in \mathcal{T}$ using a quadratic model [8]:

$$C_g(p_g(t)) \triangleq \alpha_g \left(p_g(t)\Delta t\right)^2 + \beta_g p_g(t)\Delta t + c_g, \tag{5}$$

where α_g, β_g, and c_g are positive constants.

Renewable DERs such as PVs and WTs are not dispatchable and their output is dependant on the availability of the primary sources (i.e., sun irradiance or wind). Therefore, forecasting is required in order to consider them in the energy management optimization. Methods for PV forecasting [16] and WT forecasting [17] can be utilized.

C. DS Model

We consider batteries as the DS units in the microgrid. For a given battery $b \in \mathcal{B}$, we denote its complex power by $s_b(t) \triangleq p_b(t) + \mathbf{i}q_b(t)$, where $p_b(t)$ is the active power (positive for charging and negative for discharging) and $q_b(t)$ is the reactive power. Let $E_b(t)$ denote the energy stored in the battery at time t. A given battery $b \in \mathcal{B}$ can be modeled by the following constraints:

$$p_b^{\min} \le p_b(t) \le p_b^{\max}, \forall t \in \mathcal{T}, \tag{6}$$

$$q_b^{\min} \le q_b(t) \le q_b^{\max}, \forall t \in \mathcal{T}, \tag{7}$$

$$E_b(t+1) = E_b(t) + p_b(t)\Delta t, \forall t \in \mathcal{T}, \tag{8}$$

$$E_b^{\min} \le E_b(t) \le E_b^{\max}, \forall t \in \mathcal{T}, \tag{9}$$

$$E_b(T) \ge E_b^e, \tag{10}$$

where $p_b^{\max}$ is the maximum charging rate, $-p_b^{\min}$ is the maximum discharging rate, $q_b^{\min}$ and $q_b^{\max}$ are the minimum and maximum reactive power, respectively, $E_b^{\min}$ and $E_b^{\max}$ are the minimum and maximum allowed energy stored in the battery, respectively, and E_b^e is the minimum energy that the battery should maintain at the end of the scheduling horizon.

The cost of operating a given battery $b \in \mathcal{B}$ is modelled as [18]:

$$\begin{aligned} C_b(\mathbf{p}_b) \triangleq\ & \alpha_b \sum_{t \in \mathcal{T}} p_b(t)^2 - \beta_b \sum_{t=0}^{T-2} p_b(t+1)p_b(t) \\ & + \gamma_b \sum_{t \in \mathcal{T}} \left(\min(E_b(t) - \delta_b E_b^{\max}, 0)\right)^2 + c_b, \end{aligned} \tag{11}$$

where $\mathbf{p}_b$ is the charging/discharging vector $\mathbf{p}_b \triangleq (p_b(t), t \in \mathcal{T})$, α_b, β_b, γ_b, δ_b, and c_b are positive constants.

The above function is convex when $\alpha_b > \beta_b$. This cost function captures the damages to the battery by the charging and discharging operations. The three terms in the function penalize the fast charging, the charging/discharging cycles, and the deep discharging, respectively. We choose $\delta_b = 0.2$.

D. Load Model

We consider a DSM in the microgrid, where the loads can be shedded in response to the supply condition. For each load $l \in \mathcal{L}$, we denote its complex power by $s_l(t) \triangleq p_l(t) + \mathbf{i}q_l(t)$ and it is bounded by:

$$p_l^{\min}(t) \le p_l(t) \le p_l^{\max}(t), \forall t \in \mathcal{T}, \tag{12}$$

$$q_l^{\min}(t) \le q_l(t) \le q_l^{\max}(t), \forall t \in \mathcal{T}, \tag{13}$$

where $p_l^{\min}(t)$ and $p_l^{\max}(t)$ are the minimum and maximum active power, respectively, and $q_l^{\min}(t)$ and $q_l^{\max}(t)$ are the minimum and maximum reactive power, respectively.

For each load $l \in \mathcal{L}$, we define a demand vector denoted by $\mathbf{p}_l \triangleq (p_l(t), t \in \mathcal{T})$ and a cost function $C_l(\mathbf{p}_l)$ which measures the dissatisfaction of the customer in the DSM using the demand schedule $\mathbf{p}_l$. The cost function is dependent on the shedded load and can be defined as:

$$C_l(\mathbf{p}_l) \triangleq \sum_{t \in \mathcal{T}} \alpha_l (p_l(t) - p_l^f(t))^2 + c_l, \tag{14}$$

where $p_l^f(t)$ is the forecasted load and α_l and c_l are positive constants.

Note that we consider only load shedding here. Our model can be easily extended to include load shifting and detailed load models (for example, the appliance models in [18]).

E. Distribution Network Model

A distribution network can be modeled as a connected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where each node $i \in \mathcal{N}$ represents a bus and each link in $\mathcal{E}$ represents a branch (line or transformer). We denote a link by $(i,j) \in \mathcal{E}$. Power distribution networks are typically radial and the graph $\mathcal{G}$ becomes a tree for radial distribution systems. We index the buses in $\mathcal{N}$ by $i = 0, 1, \ldots, n$, and bus 0 denotes the feeder which has a fixed voltage and flexible power injection.

For each link $(i,j) \in \mathcal{E}$, let $z_{ij} \triangleq r_{ij} + \mathbf{i}x_{i,j}$ be the complex impedance of the branch, $I_{ij}(t)$ be complex current from buses i to j, and $S_{ij}(t) \triangleq P_{ij}(t) + \mathbf{i}Q_{ij}(t)$ be the complex power flowing from buses i to j.

For each bus $i \in \mathcal{N}$, let $V_i(t)$ be the complex voltage at bus i and $s_i(t) \triangleq p_i(t) + \mathbf{i}q_i(t)$ be the net load which is the load minus the generation at bus i. Each bus $i \in \mathcal{N} \setminus \{0\}$ is connected to a subset of DG units $\mathcal{G}_i$, DS units $\mathcal{B}_i$, and loads $\mathcal{L}_i$. The net load at each bus i satisfies:

$$s_i(t) = s_{li}(t) + s_{bi}(t) - s_{gi}(t),\ \forall i \in \mathcal{N} \setminus \{0\}, \forall t \in \mathcal{T}, \tag{15}$$

where $s_{li}(t) \triangleq \sum_{l \in \mathcal{L}_i} s_l(t)$, $s_{bi}(t) \triangleq \sum_{b \in \mathcal{B}_i} s_b(t)$, and $s_{gi} \triangleq \sum_{g \in \mathcal{G}_i} s_g(t)$.

The steady-state power flows in a given distribution network $\mathcal{G}$ can be modeled using the branch flow model [19]: $\forall (i,j) \in \mathcal{E}, \forall t \in \mathcal{T}$,

$$p_j(t) = P_{ij}(t) - r_{ij}\ell_{ij}(t) - \sum_{k:(j,k) \in \mathcal{E}} P_{jk}(t), \tag{16}$$

$$q_j(t) = Q_{ij}(t) - x_{ij}\ell_{ij}(t) - \sum_{k:(j,k) \in \mathcal{E}} Q_{jk}(t), \tag{17}$$

$$v_j(t) = v_i(t) - 2\left(r_{ij}P_{ij}(t) + x_{ij}Q_{ij}(t)\right) + (r_{ij}^2 + x_{ij}^2)\ell_{ij}(t), \tag{18}$$

$$\ell_{ij}(t) = \frac{P_{ij}(t)^2 + Q_{ij}(t)^2}{v_i(t)}, \tag{19}$$

where $\ell_{ij}(t) \triangleq |I_{ij}(t)|^2$ and $v_i(t) \triangleq |V_i(t)|^2$.

Equations (16)–(19) define a system of equations in the variables $(\mathbf{P}(t), \mathbf{Q}(t), \mathbf{v}(t), \mathbf{l}(t), \mathbf{s}(t))$, where $\mathbf{P}(t) \triangleq (P_{ij}(t), (i,j) \in \mathcal{E})$, $\mathbf{Q}(t) \triangleq (Q_{ij}(t), (i,j) \in \mathcal{E})$, $\mathbf{v}(t) \triangleq (v_i(t), i \in \mathcal{N} \setminus \{0\})$, $\mathbf{l}(t) \triangleq (\ell_{ij}(t), (i,j) \in \mathcal{E})$, and $\mathbf{s}(t) \triangleq (s_i(t), i \in \mathcal{N} \setminus \{0\})$. The phase angles of the voltages and the currents are not included. But they can be uniquely determined for radial systems [19].

F. Energy Management

We consider the voltage tolerance constraints in the microgrid:

$$V_i^{\min} \leq |V_i(t)| \leq V_i^{\max},\ \forall i \in \mathcal{N} \setminus \{0\}, \forall t \in \mathcal{T}, \quad (20)$$

where $V_i^{\min}$ and $V_i^{\max}$ correspond to the minimum and maximum allowed voltages, respectively.

The net power injected to the microgrid from the main grid is given by:

$$s_0(t) = \sum_{j:(0,j)\in\mathcal{E}} s_{0j}(t), \forall t \in \mathcal{T}. \quad (21)$$

If the microgrid is operated in islanded mode, then $s_0(t) = 0$. If the microgrid is operated in grid-connected mode, then $s_0(t)$ is the net complex power traded between the microgrid and the main grid.

We model the cost of energy purchase from the main grid as:

$$C_0(t, p_0(t)) \triangleq \rho(t) p_0(t) \Delta t, \quad (22)$$

where $\rho(t)$ is the market energy price. Note that $p_0(t)$ can be negative, meaning that the microgrid can sell its surplus power to the main grid.

The objective of the energy management in the microgrid is to (i) minimize the cost of generation, the cost of energy storage, and the cost of energy purchase from the main grid, and (ii) minimize the dissatisfactions of the customers in the DSM, and (iii) minimize the power losses subject to the DER constraints, the load constraints, the power flow constraints, and the system operational constraints.

We define $\mathbf{P} \triangleq (\mathbf{P}(t), t \in \mathcal{T})$, $\mathbf{Q} \triangleq (\mathbf{Q}(t), t \in \mathcal{T})$, $\mathbf{v} \triangleq (\mathbf{v}(t), t \in \mathcal{T})$, $\mathbf{l} \triangleq (\mathbf{l}(t), t \in \mathcal{T})$, $\mathbf{s}_g \triangleq (s_g(t), t \in \mathcal{T})$, $\mathbf{s}_b \triangleq (s_b(t), t \in \mathcal{T})$, $\mathbf{s}_l \triangleq (s_l(t), t \in \mathcal{T})$, $\mathbf{s} \triangleq (\mathbf{s}_g, \mathbf{s}_b, \mathbf{s}_l, g \in \mathcal{G}, b \in \mathcal{B}, l \in \mathcal{L})$, and $C_g(\mathbf{p}_g) \triangleq \sum_{t\in\mathcal{T}} C_g(p_g(t))$. The energy management in the microgrid can be formulated as an OPF problem:

OPF:

$$\begin{aligned}\min_{\mathbf{P},\mathbf{Q},\mathbf{v},\mathbf{l},\mathbf{s}} \quad & \xi_g \sum_{g\in\mathcal{G}} C_g(\mathbf{p}_g) + \xi_b \sum_{b\in\mathcal{B}} C_b(\mathbf{p}_b) + \xi_l \sum_{l\in\mathcal{L}} C_l(\mathbf{p}_l) \\ & + \xi_0 \sum_{t\in\mathcal{T}} C_0(t, p_0(t)) + \xi_p \sum_{t\in\mathcal{T}} \sum_{(i,j)\in\mathcal{E}} r_{ij} \ell_{ij}(t) \\ \text{s.t.} \quad & (1)-(4), (6)-(10), (12), (13), (15)-(21),\end{aligned}$$

where ξ_g, ξ_b, ξ_l, ξ_0, and ξ_p are the parameters to trade off among different cost minimizations.

III. Distributed EMS

The previous OPF problem is non-convex due to the quadratic equality constraint in (19) and is NP-hard to solve in general [10]. We therefore relax them to inequalities:

$$\ell_{ij}(t) \geq \frac{P_{ij}(t)^2 + Q_{ij}(t)^2}{v_i(t)},\ \forall (i,j) \in \mathcal{E}, \forall t \in \mathcal{T}. \quad (23)$$

We then consider the following convex relaxation of OPF:

OPF-r:

$$\begin{aligned}\min_{\mathbf{P},\mathbf{Q},\mathbf{v},\mathbf{l},\mathbf{s}} \quad & \xi_g \sum_{g\in\mathcal{G}} C_g(\mathbf{p}_g) + \xi_b \sum_{b\in\mathcal{B}} C_b(\mathbf{p}_b) + \xi_l \sum_{l\in\mathcal{L}} C_l(\mathbf{p}_l) \\ & + \xi_0 \sum_{t\in\mathcal{T}} C_0(t, p_0(t)) + \xi_p \sum_{t\in\mathcal{T}} \sum_{(i,j)\in\mathcal{E}} r_{ij} \ell_{ij}(t) \\ \text{s.t.} \quad & (1)-(4), (6)-(10), (12), (13), (15)-(18), \\ & (20)-(21), (23).\end{aligned}$$

If the equality in (23) is attained in the solution to OPF-r, then it is also an optimal solution to OPF. The sufficient conditions under which the relaxation is exact have been exploited in previous works [12]–[14]. In this paper, we assume that the sufficient conditions specified in [14] hold for the microgrid and thus we focus on solving the OPF-r problem.

The above OPF-r problem is a centralized optimization problem. In order to design an efficient, scalable, and privacy-preserving EMS, we propose a distributed algorithm to solve the OPF-r problem using the predictor corrector proximal multiplier (PCPM) algorithm [20].

Initially set $k \leftarrow 0$. The LCs of the DERs and loads set their initial schedules randomly and communicate them to the MGCC. In the meantime, the MGCC randomly chooses the initial $s_i^k(t) \triangleq p_i^k(t) + \mathbf{i} q_i^k(t)$ and two virtual control signals $\{\mu_i^k(t)\}_{t\in\mathcal{T}}$, $\{\lambda_i^k(t)\}_{t\in\mathcal{T}}$ for each bus $i \in \mathcal{N} \setminus \{0\}$.

At the beginning of the k-th step, the MGCC sends two control signals $\hat{\mu}_i^k(t) \triangleq \mu_i^k(t) + \gamma\left(p_{li}^k(t) + p_{bi}^k(t) - p_{gi}^k(t) - p_i^k(t)\right)$ and $\hat{\lambda}_i^k(t) \triangleq \lambda_i^k(t) + \gamma\left(q_{li}^k(t) + q_{bi}^k(t) - q_{gi}^k(t) - q_i^k(t)\right)$ to the LCs of the DERs and the loads connected to bus i, where γ is a positive constant. Then,

- The LC of each DG unit solves the following problem:
EMS-LC(DG):

$$\begin{aligned}\min_{\mathbf{s}_g} \quad & \xi_g C_g(\mathbf{p}_g) + (\hat{\boldsymbol{\mu}}_i^k)^T \mathbf{p}_g + (\hat{\boldsymbol{\lambda}}_i^k)^T \mathbf{q}_g \\ & + \frac{1}{2\gamma} ||\mathbf{p}_g - \mathbf{p}_g^k||^2 + \frac{1}{2\gamma} ||\mathbf{q}_g - \mathbf{q}_g^k||^2 \\ \text{s.t.} \quad & (1)-(4),\end{aligned}$$

where $\hat{\boldsymbol{\mu}}_i^k \triangleq (\hat{\mu}_i^k(t), t \in \mathcal{T})$ and $\hat{\boldsymbol{\lambda}}_i^k \triangleq (\hat{\lambda}_i^k(t), t \in \mathcal{T})$. The optimal $\mathbf{s}_g^*$ is set as $\mathbf{s}_g^{k+1}$.
- The LC of each DS unit solves the following problem:
EMS-LC(DS):

$$\begin{aligned}\min_{\mathbf{s}_b} \quad & \xi_b C_b(\mathbf{p}_b) + (\hat{\boldsymbol{\mu}}_i^k)^T \mathbf{p}_b + (\hat{\boldsymbol{\lambda}}_i^k)^T \mathbf{q}_b \\ & + \frac{1}{2\gamma} ||\mathbf{p}_b - \mathbf{p}_b^k||^2 + \frac{1}{2\gamma} ||\mathbf{q}_b - \mathbf{q}_b^k||^2 \\ \text{s.t.} \quad & (6)-(10).\end{aligned}$$

The optimal $\mathbf{s}_b^*$ is set as $\mathbf{s}_b^{k+1}$.
- The LC of each load solves the following problem:
EMS-LC(Load):

$$\begin{aligned}\min_{\mathbf{s}_l} \quad & \xi_l C_l(\mathbf{p}_l) + (\hat{\boldsymbol{\mu}}_i^k)^T \mathbf{p}_l + (\hat{\boldsymbol{\lambda}}_i^k)^T \mathbf{q}_l \\ & + \frac{1}{2\gamma} ||\mathbf{p}_l - \mathbf{p}_l^k||^2 + \frac{1}{2\gamma} ||\mathbf{q}_l - \mathbf{q}_l^k||^2 \\ \text{s.t.} \quad & (12), (13).\end{aligned}$$

Algorithm 1 - The Proposed Distributed EMS.

initialization $k \leftarrow 0$. The LCs set the initial schedules randomly and return them to the MGCC. The MGCC sets the initial $\mu_i^k(t)$, $\lambda_i^k(t)$ and the initial $s_i^k(t)$ randomly.
repeat
  The MGCC updates $\hat{\mu}_i^k(t)$ and $\hat{\lambda}_i^k(t)$ and sends two control signals $\hat{\boldsymbol{\mu}}_i^k$ and $\hat{\boldsymbol{\lambda}}_i^k$ to the LCs connected to bus i.
  The LC at each DER and each load calculates a new schedule by solving the corresponding EMS-LC problem.
  The MGCC computes a new $\mathbf{s}^{k+1}(t)$ for each time $t \in \mathcal{T}$ by solving the EMS-MGCC problem.
  The LC communicates the new schedule to the MGCC.
  The MGCC updates $\mu_i^{k+1}(t)$ and $\lambda_i^{k+1}(t)$.
  $k \leftarrow k+1$.
until convergence

The optimal $\mathbf{s}_l^*$ is set as $\mathbf{s}_l^{k+1}$.

- The MGCC solves the following problem for each time $t \in \mathcal{T}$:
  EMS-MGCC:

$$\begin{aligned}\min_{\substack{\mathbf{P}(t),\mathbf{Q}(t),\\ \mathbf{v}(t),\mathbf{l}(t),\mathbf{s}(t)}} \quad & \xi_0 C_0(t,p_0(t)) + \xi_p \sum_{(i,j)\in\mathcal{E}} r_{ij}\ell_{ij}(t) \\ & -(\hat{\boldsymbol{\mu}}^k(t))^T\mathbf{p}(t) - (\hat{\boldsymbol{\lambda}}^k(t))^T\mathbf{q}(t) \\ & + \frac{1}{2\gamma}||\mathbf{p}(t)-\mathbf{p}^k(t)||^2 + \frac{1}{2\gamma}||\mathbf{q}(t)-\mathbf{q}^k(t)||^2 \\ \text{s.t.} \quad & (16)-(18),(20)-(21),(23),\end{aligned}$$

  where $\hat{\boldsymbol{\mu}}^k(t) \triangleq (\hat{\mu}_i^k(t), i \in \mathcal{N}\setminus\{0\})$ and $\hat{\boldsymbol{\lambda}}^k(t) \triangleq (\hat{\lambda}_i^k(t), i \in \mathcal{N}\setminus\{0\})$. The optimal $\mathbf{s}^*(t)$ is set as $\mathbf{s}^{k+1}(t)$.

At the end of the k-th step, the LCs communicate their new schedules $\mathbf{s}_l^{k+1}$, $\mathbf{s}_g^{k+1}$, and $\mathbf{s}_b^{k+1}$ to the MGCC and the MGCC updates $\mu_i^{k+1}(t) \triangleq \mu_i^k(t) + \gamma\left(p_{li}^{k+1}(t) + p_{bi}^{k+1}(t) - p_{gi}^{k+1}(t) - p_i^{k+1}(t)\right)$ and $\lambda_i^{k+1}(t) \triangleq \lambda_i^k(t) + \gamma\left(q_{li}^{k+1}(t) + q_{bi}^{k+1}(t) - q_{gi}^{k+1}(t) - q_i^{k+1}(t)\right)$ for all $i \in \mathcal{N}\setminus\{0\}$ and all $t \in \mathcal{T}$. Set $k \leftarrow k+1$, and repeat the process until convergence.

When γ is small enough, the above algorithm will converge to the optimal solution of OPF-r which is also the optimal solution of OPF and $\left(p_{li}^k(t) + p_{bi}^k(t) - p_{gi}^k(t) - p_i^k(t)\right)$ and $\left(q_{li}^k(t) + q_{bi}^k(t) - q_{gi}^k(t) - q_i^k(t)\right)$ will converge to zero [20].

In the proposed distributed EMS, the private information of the DERs and the loads is stored at the LC where the EMS-LC problem is solved locally. The MGCC solves the EMS-MGCC problem using the system information, including the topology, the power losses, etc. The information exchanged between the MGCC and the LCs include only the control signals and the schedules. Therefore, the privacy of the DERs (i.e., production costs and constraints) and the loads (i.e., customer preferences and constraints) are both preserved by the proposed EMS.

IV. Case Study

As one demonstration, we apply the proposed EMS to a real microgrid in Guangdong Province, China as shown in Fig. 2. The numbers under the DERs and the loads in the figure correspond to the maximum power. We set the cost function of the diesel generation as $C_g(p_g(t)) \triangleq 0.1(p_g(t)\Delta t)^2 + 0.7(p_g(t)\Delta t)$. The capacity of the BESS $E_b^{\max}$ is 3MWh and $E_b^{\min}$ is chosen to be 0.1MWh. We set $E_b(0) = 1.5$MWh and $E_b^e = 1.0$MWh. The parameters in the cost function of the battery are chosen as $\alpha_b = 1$, $\beta_b = 0.75$, and $\gamma_b = 0.5$. The cost function of the loads is chosen to be $C_l(\mathbf{p}_l) \triangleq \sum_{t\in\mathcal{T}} 10(p_l(t) - p_l^f(t))^2$. We assume that the DSM is able to shed a certain percentage of the forecasted load. The maximum load shedding percentage is chosen randomly from the range $[0\%, 20\%]$. Perfect forecasting of the PV, the WT, and the loads is assumed. The day-ahead energy price $\rho(t)$ is given by Fig 3. The voltage tolerances are set to be $[0.95V_r, 1.05V_r]$, where V_r is the rated voltage. The parameters in the algorithm are chosen as $\xi_g = 1$, $\xi_b = 0.01$, $\xi_l = 1$, $\xi_0 = 1$, $\xi_p = 0.01$, and $\gamma = 0.5$.

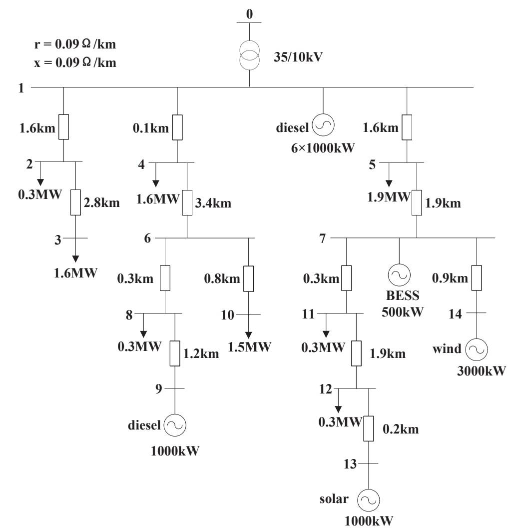

Fig. 2. The topology of the microgrid.

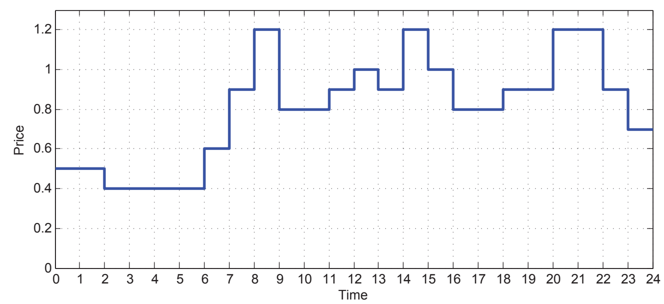

Fig. 3. Energy price of a day.

The day-ahead schedules produced by the proposed EMS in islanded and grid-connected mode are shown in Fig. 4 and Fig. 5, respectively. From Fig. 4, we can see that the total diesel generation changes in the same trend as the total load in islanded mode. This is because diesel is the main source of generation in the microgird. We can also observe the charging/discharging cycles of the battery in the figure. The battery is charged when the renewable power is high and discharged when it is low, serving as the storage for renewables in the

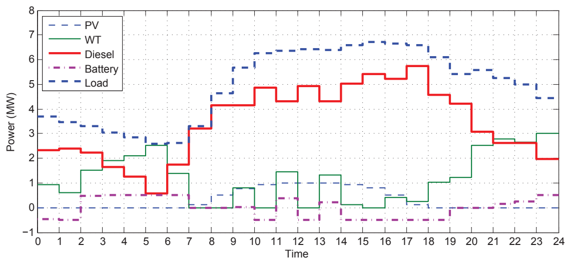

Fig. 4. The output schedules in islanded mode.

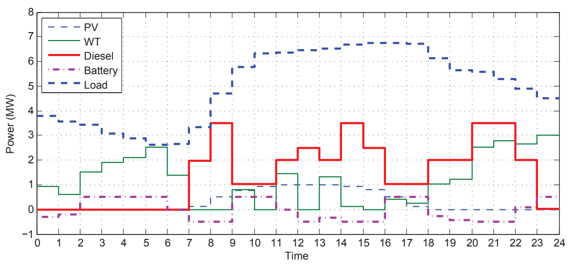

Fig. 5. The output schedules in grid-connected mode.

microgrid. By comparing Fig. 4 with Fig. 5, it can be easily seen that the total diesel generation is decreased significantly in grid-connected mode as the microgrid can import energy from the main grid. The battery in grid-connected mode is the storage for not only renewables but also the cheap power from the main grid. It is also charged when the energy price is low and discharged when the energy price is high, making profits for the microgrid.

Fig. 6 shows the value of the objective function over iterations at $t = 10$ in grid-connected mode. We can see that our proposed algorithm converges fast. For the simulations, we also verify that the solution to the centralized OPF-r problem is the same as the solution of the distributed algorithm. We further verify that the equality in (23) is attained in the optimal solution to OPF-r, i.e., OPF-r is an exact relaxation of OPF.

V. Conclusions

In this paper, we propose a distributed EMS for the optimal operation of microgrids. Compared with the existing distributed approaches, our proposed EMS considers the underlying power distribution network and the associated constraints. Specifically, we formulate microgrid energy management as an OPF problem and propose a distributed EMS where the MGCC and the LCs jointly compute an optimal schedule. As one demonstration, we apply the proposed EMS to a real microgrid in China. The simulation results show that the proposed EMS is effective in both islanded and grid-connected mode and the proposed algorithm converges fast.

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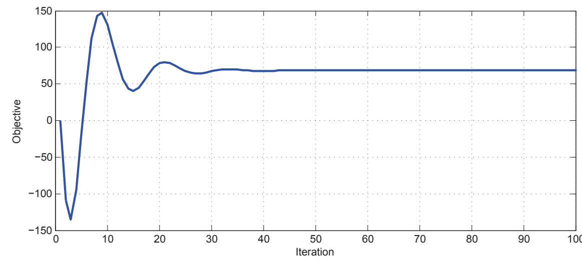

Fig. 6. The value of the objective function over iterations.

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